INFLUENCE OF ROUGHNESS ON THE HYDRODYNAMIC DESCRIPTION OF A GRANULAR GAS

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In collaboration with

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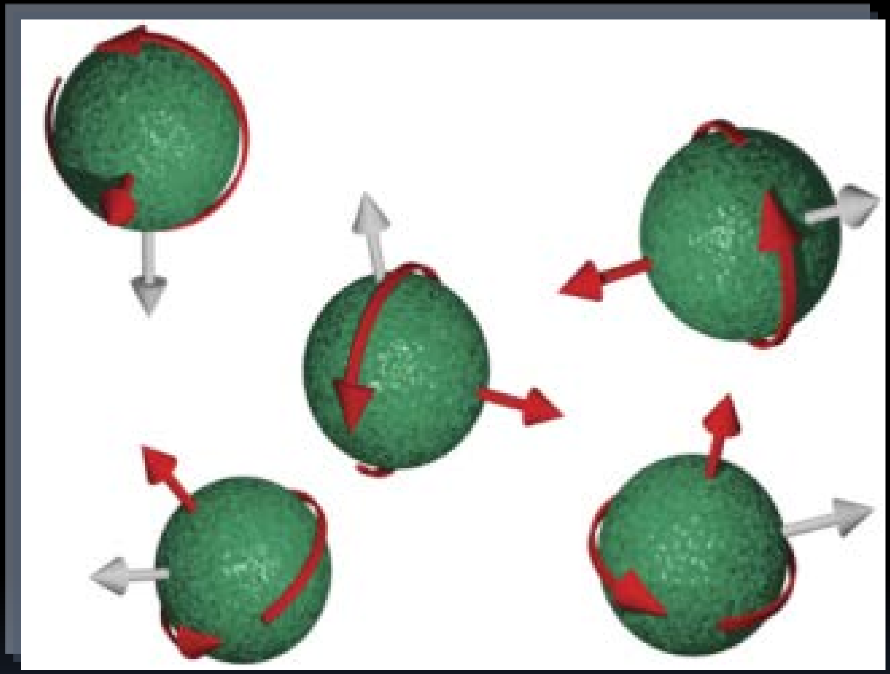
- V. Garzó (Badajoz, Spain)
- G. M. Kremer (Curitiba, Brazil)

Simple model of a granular gas: A *collection* of *inelastic* *rough* hard spheres



This model unveils an inherent breakdown of energy equipartition in granular fluids, even in homogeneous and isotropic states

Material parameters:

- Mass m
- Diameter σ
- Moment of inertia I $(\kappa=4I/m\sigma^2)$
- Coefficient of normal restitution α
- Coefficient of tangential restitution β
- $\alpha=1$ for perfectly elastic particles
- $\beta=-1$ for perfectly smooth particles
- $\beta=+1$ for perfectly rough particles

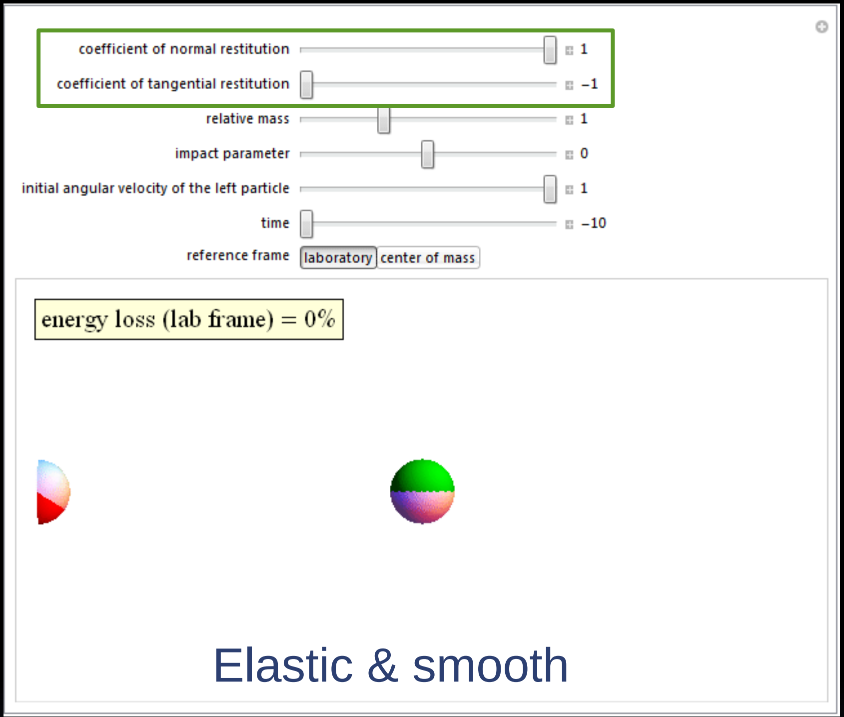
Energy collisional loss

$$E_{ij} = \frac{1}{2}mv_i^2 + \frac{1}{2}mv_j^2 + \frac{1}{2}I\omega_i^2 + \frac{1}{2}I\omega_j^2$$

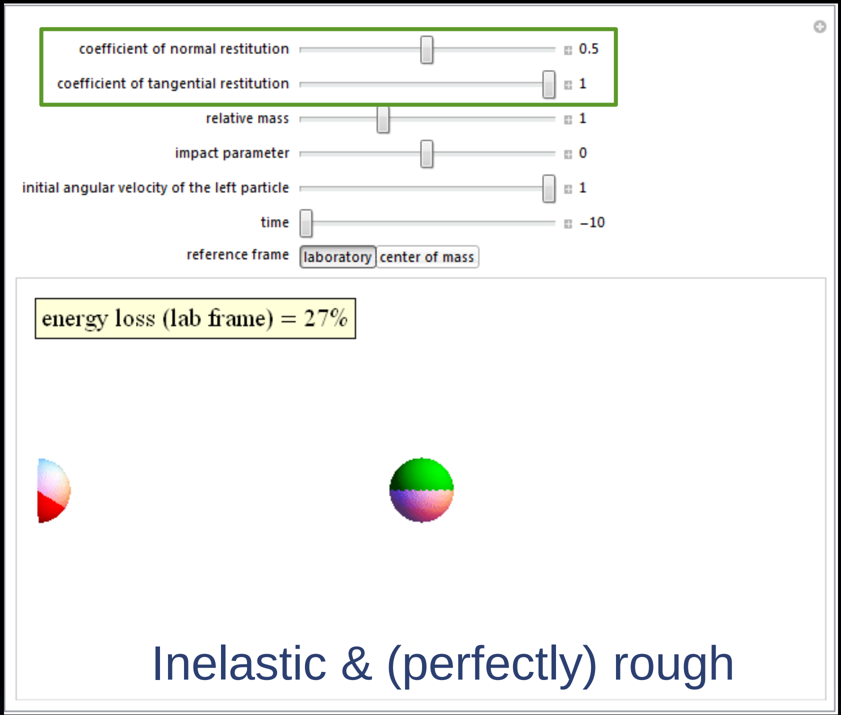
$$\begin{aligned} E'_{ij} - E_{ij} &= -(1-\alpha^2)\times\cdots \\ &\quad -(1-\beta^2)\times\cdots \end{aligned}$$

Energy is conserved *only* if the spheres are

- elastic (α=1) and
- either
  - perfectly smooth (β=-1) or
  - perfectly rough (β=+1)



Elastic & smooth



Inelastic & (perfectly) rough

http://demonstrations.wolfram.com/InelasticCollisionsOfTwoRoughSpheres/

Outline of the talk

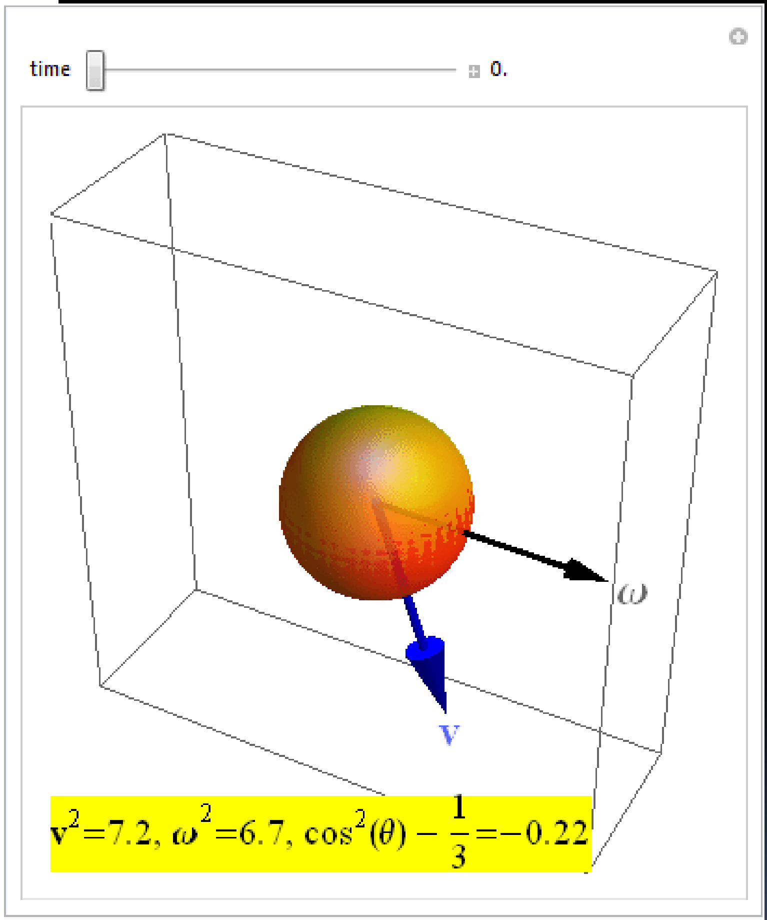
- 1. Homogeneous cooling state. Velocity cumulants.
- 2. Navier-Stokes-Fourier transport coefficients.

F. Vega Reyes, A. S., and G. M. Kremer, Phys. Rev. E **89**, 020202(R) (2014)

Francisco Vega Reyes

Gilberto M. Kremer

Granular temperatures, kurtoses, and correlations



translational temperature: $\langle v^2\rangle = \dfrac{3T_t}{m}$

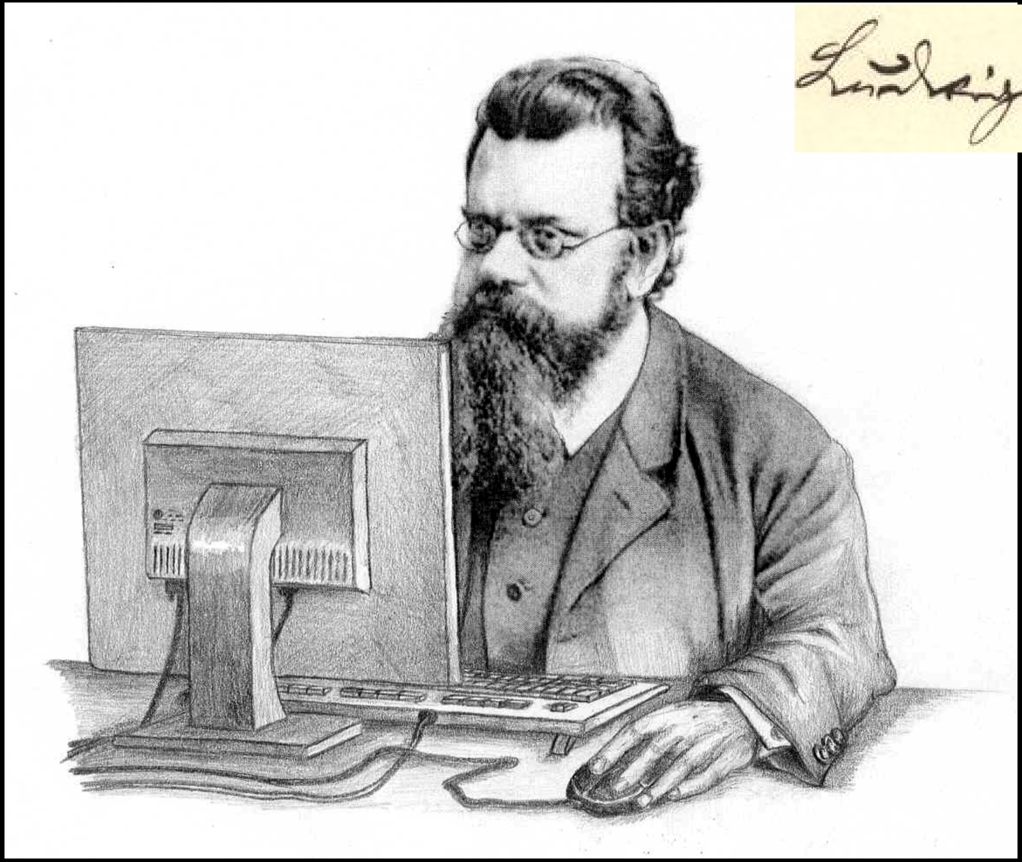
rotational temperature: $\langle \omega^2\rangle = \dfrac{3T_r}{I}$

translational kurtosis: $\langle v^4\rangle = \dfrac{5}{3}\langle v^2\rangle^2\left(1+a_{20}^{(0)}\right)$

rotational kurtosis: $\langle \omega^4\rangle = \dfrac{5}{3}\langle \omega^2\rangle^2\left(1+a_{02}^{(0)}\right)$

scalar correlations: $\langle v^2\omega^2\rangle = \langle v^2\rangle\langle \omega^2\rangle\left(1+a_{11}^{(0)}\right)$

angular correlations: $\langle(\mathbf{v}\cdot\boldsymbol{\omega})^2\rangle - \dfrac{1}{3}\langle v^2\omega^2\rangle \propto a_{00}^{(1)}$



(1844-1906)

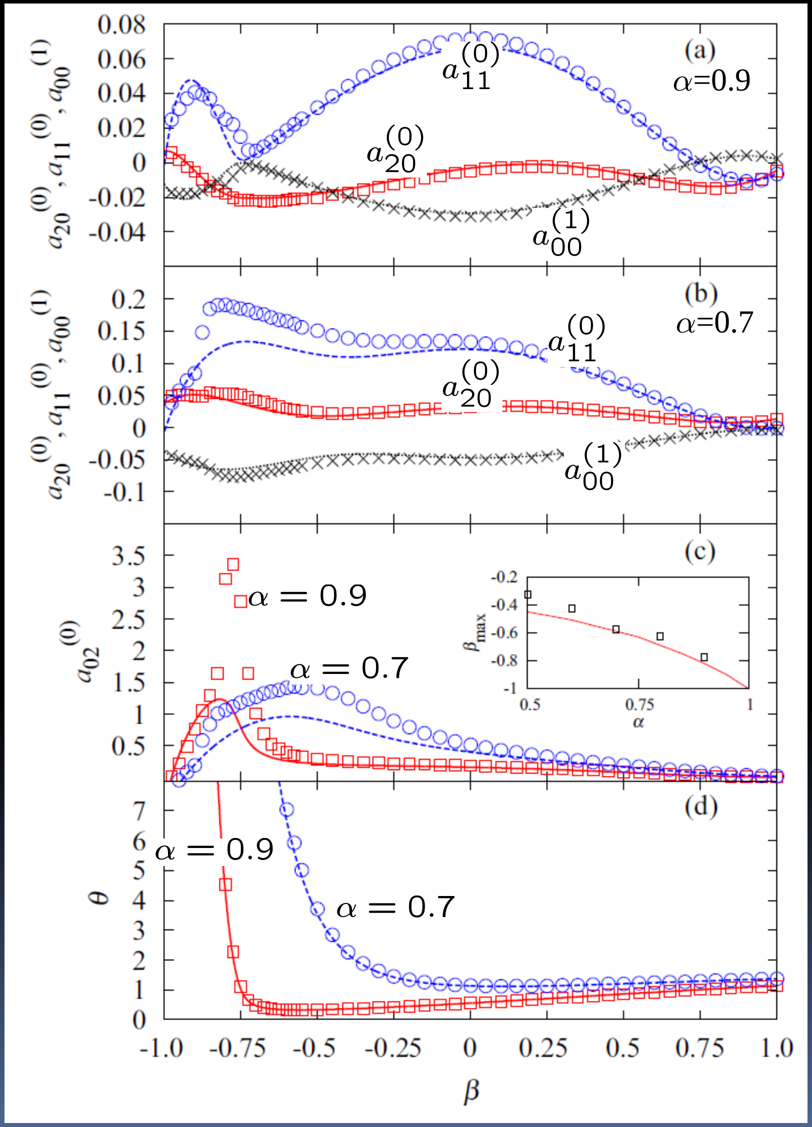
(Cartoon by Bernhard Reischl, University of Vienna)

Boltzmann equation:

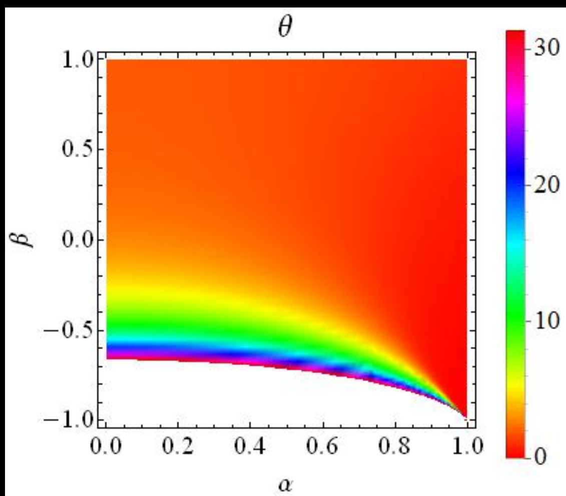
$$\partial_t f(\mathbf{r},\mathbf{v},\omega,t) + \mathbf{v}\cdot\nabla f(\mathbf{r},\mathbf{v},\omega,t) = J[\mathbf{r},\mathbf{v},\omega,t|f]$$

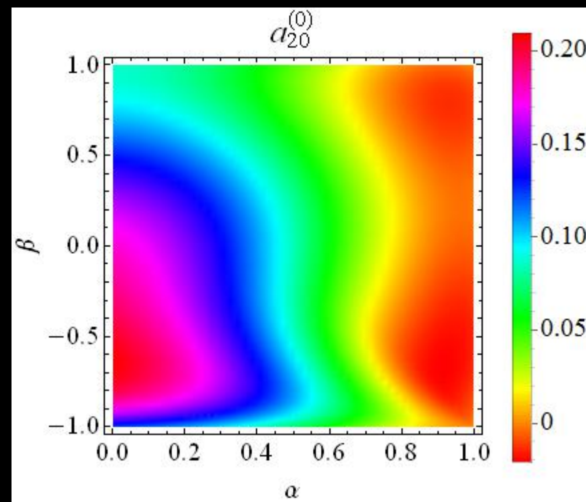
Inelastic+Rough collisions

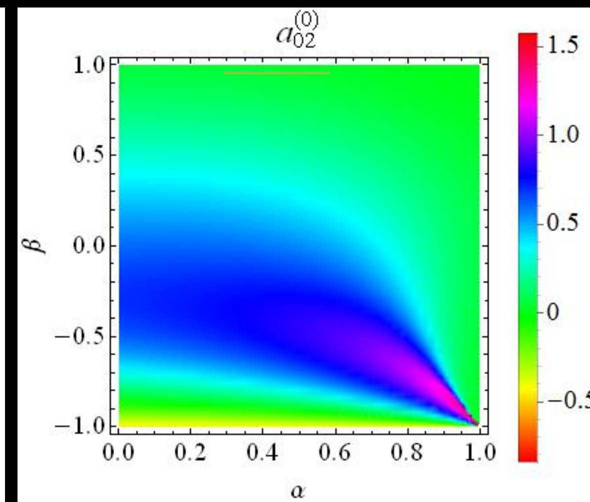
Theory (Sonine) vs simulations

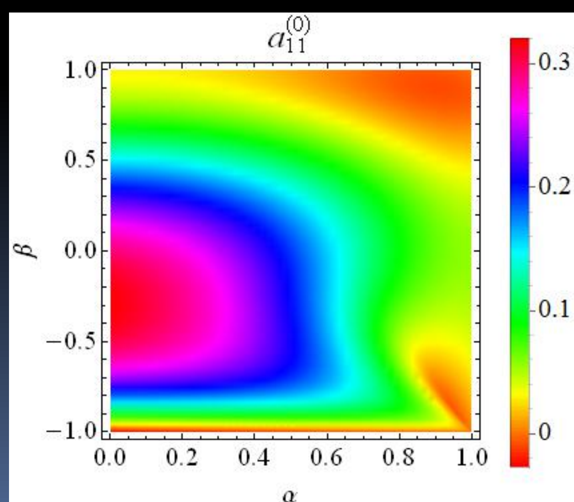


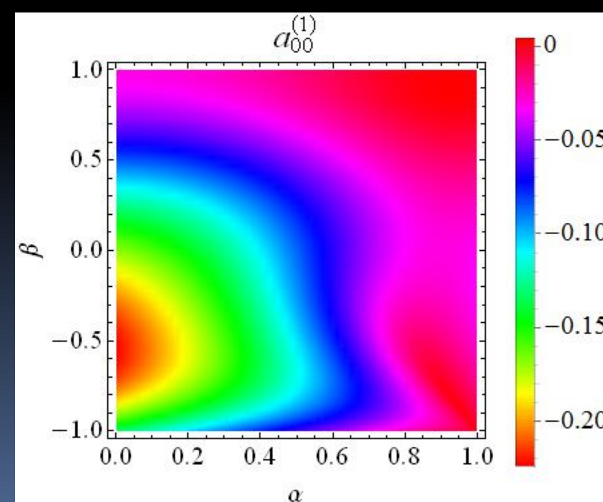
Density plots











Conclusions (Part 1)

- The linearized Sonine approximation theory provides an excellent description of the temperature ratio and the four velocity cumulants, *except* when the angular velocity kurtosis becomes large ($a_{02}^{(0)} > 0.3$).
- The cumulants are relatively small in the experimentally relevant regime $\beta > 0$.

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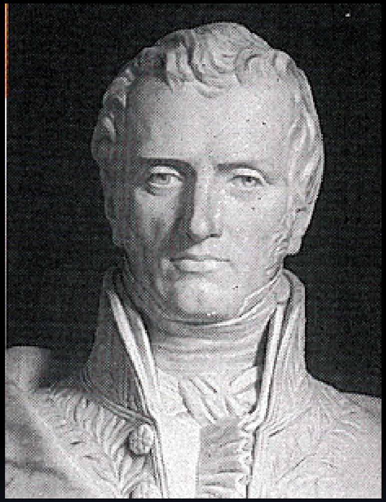
- 1. Homogeneous cooling state. Velocity cumulants.
- 2. Navier-Stokes-Fourier transport coefficients.

G. M. Kremer, A. S., and V. Garzó, Phys. Rev. E 90, 022205 (2014)

Gilberto M. Kremer

Vicente Garzó

Navier-Stokes-Fourier constitutive equations



Claude-Louis Navier
(1785-1836)

George Gabriel Stokes
(1819-1903)

Jean-Baptiste Joseph Fourier
(1768-1830)

Navier-Stokes-Fourier constitutive equations

$$P_{ij} = nT_t\delta_{ij} - \eta\left(\nabla_i u_j + \nabla_j u_i - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{u}\right) - \eta_b\delta_{ij}\nabla\cdot\mathbf{u}$$

Shear viscosity

Bulk viscosity

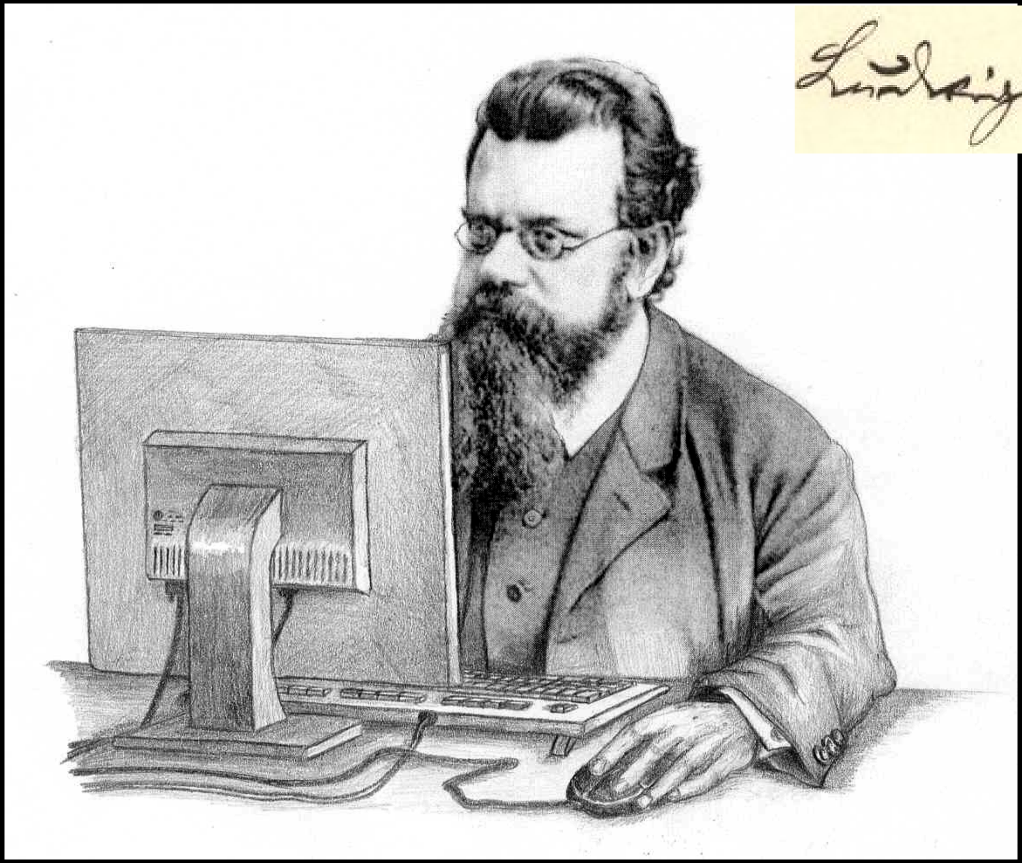
Dufour-like coefficient

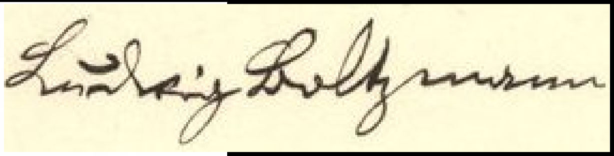
$$\mathbf{q} = -\lambda\nabla T - \mu\nabla n$$

Thermal conductivity

$$\zeta = \zeta^{(0)} - \xi\nabla\cdot\mathbf{u}$$

Cooling rate transport coefficient





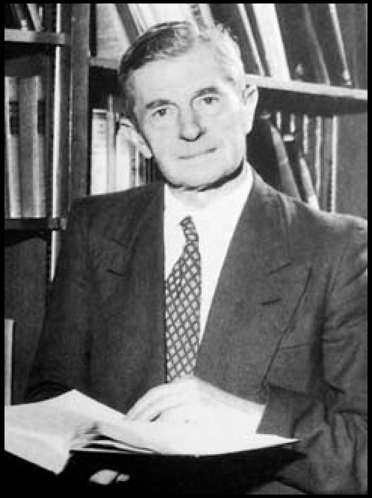
(1844-1906)

(Cartoon by Bernhard Reischl, University of Vienna)

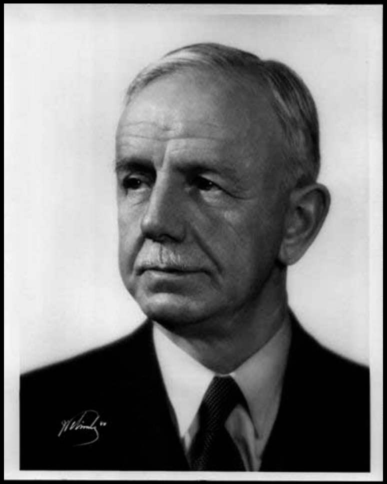
Boltzmann equation:

$$\partial_t f(\mathbf{r},\mathbf{v},\boldsymbol{\omega},t) + \boxed{\mathbf{v}\cdot\nabla f(\mathbf{r},\mathbf{v},\boldsymbol{\omega},t)} = \underbrace{J[\mathbf{r},\mathbf{v},\boldsymbol{\omega},t|f]}_{\text{Inelastic+Rough collisions}}$$

Methodology: Chapman-Enskog method



Sydney Chapman
(1888-1970)



David Enskog
(1884-1947)

$$\boxed{f = f^{(0)} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \cdots}, \quad \epsilon \sim \nabla$$

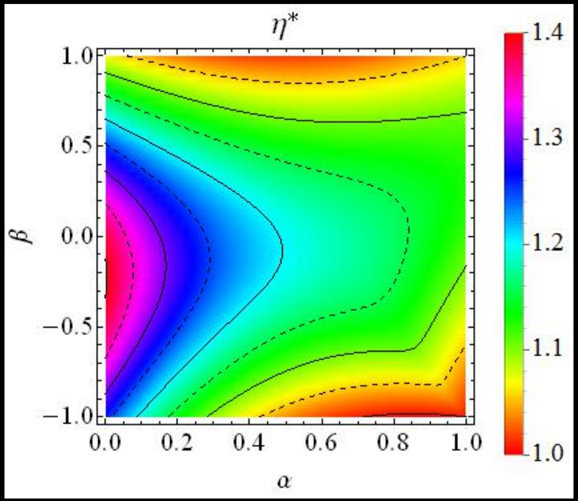
Special limiting cases

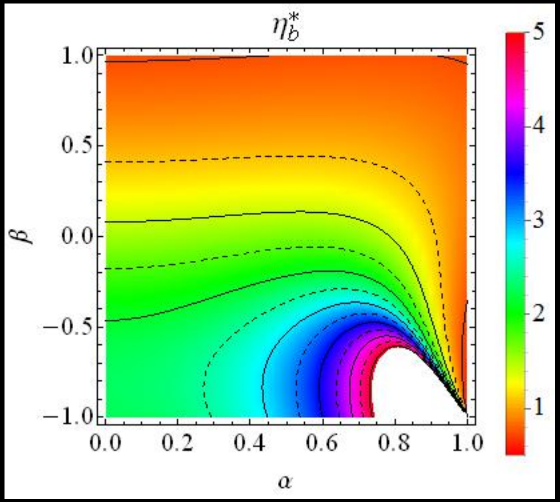
Quantity	Pure smooth ($\beta = -1$)	Quasi-smooth limit ($\beta \to -1$)	Perfectly rough and elastic ($\alpha = \beta = 1$)
η^*	$\frac{24}{(1+\alpha)(13-\alpha)}$	$\frac{24}{(1+\alpha)(19-7\alpha)}$	$\frac{6(1+\kappa)^2}{6+13\kappa}$
η_b^*	0	$\frac{8}{5(1-\alpha^2)}$	$\frac{(1+\kappa)^2}{10\kappa}$
λ^*	$\frac{64}{(1+\alpha)(9+7\alpha)}$	$\frac{48}{25(1+\alpha)}$	$\frac{12(1+\kappa)^2\left(37+151\kappa+50\kappa^2\right)}{25\left(12+75\kappa+101\kappa^2+102\kappa^3\right)}$
μ^*	$\frac{1280(1-\alpha)}{(1+\alpha)(9+7\alpha)(19-3\alpha)}$	0	0
ξ	0	0	0

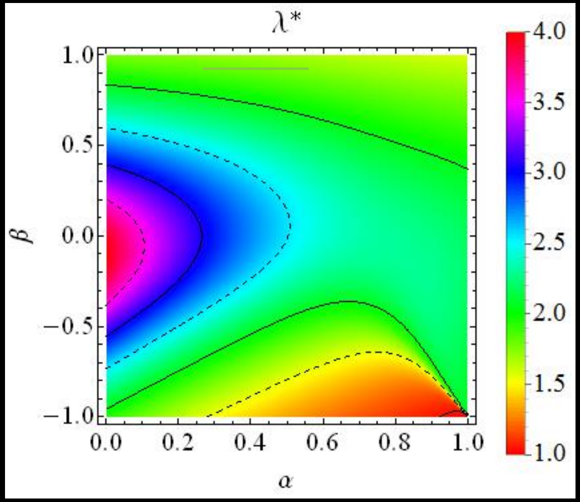
Brey, Dufty, Kim, Santos (1998) — Pure smooth

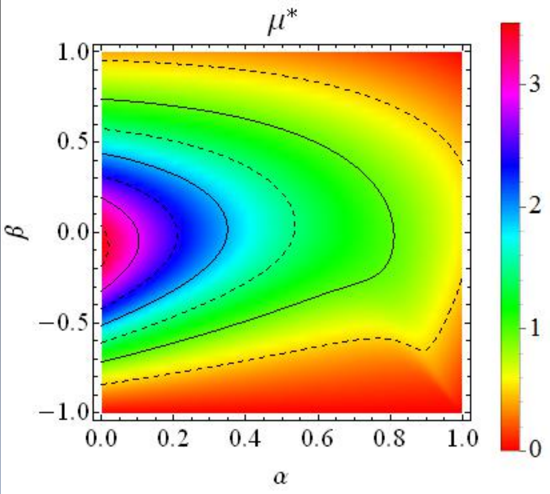
Pidduck (1922) — Perfectly rough and elastic

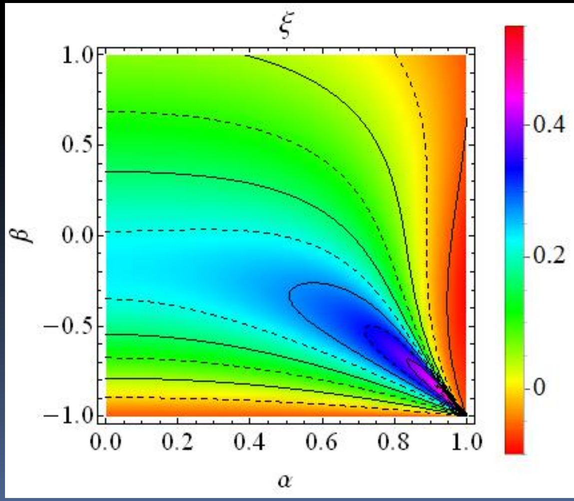
Density plots











Conclusions (Part 2)

- Roughness induces two extra transport coefficients (η_b, ξ), not present in the case of a (dilute) gas of smooth spheres.
- Typically, at fixed α the coefficients have a maximum at an intermediate value of β.
- In general, the dependence of the coefficients on α is weaker than in the case of smooth spheres.
- Future application: Stability analysis of the HCS.

Thank you for your attention!