Granular gases as a paradigm of the Mpemba effect

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Statistical Physics in eXtremadura

Universidad Carlos III de Madrid
www.uc3m.es

When I first met Luis (1970?)

Instituto de Bachillerato de Almería, 1966- 1984

Imagine we organize a party to celebrate Luis' birthday

https://www.kiddiejungle.com/parties/

http://rose-rapture.deviantart.com/art/The-Scream-Emo-fied-60135745

But …

A couple of hours before the party starts we realize no ice cubes are left in the freezer

We call a friend, who says:
"Don't panic! Use hot water!"

Is he crazy?

(Credit: Leif Parsons)

Outline

- What is the Mpemba effect?
- What is a granular gas?
- Mpemba effect in granular gases
- Mpemba effect in molecular gases

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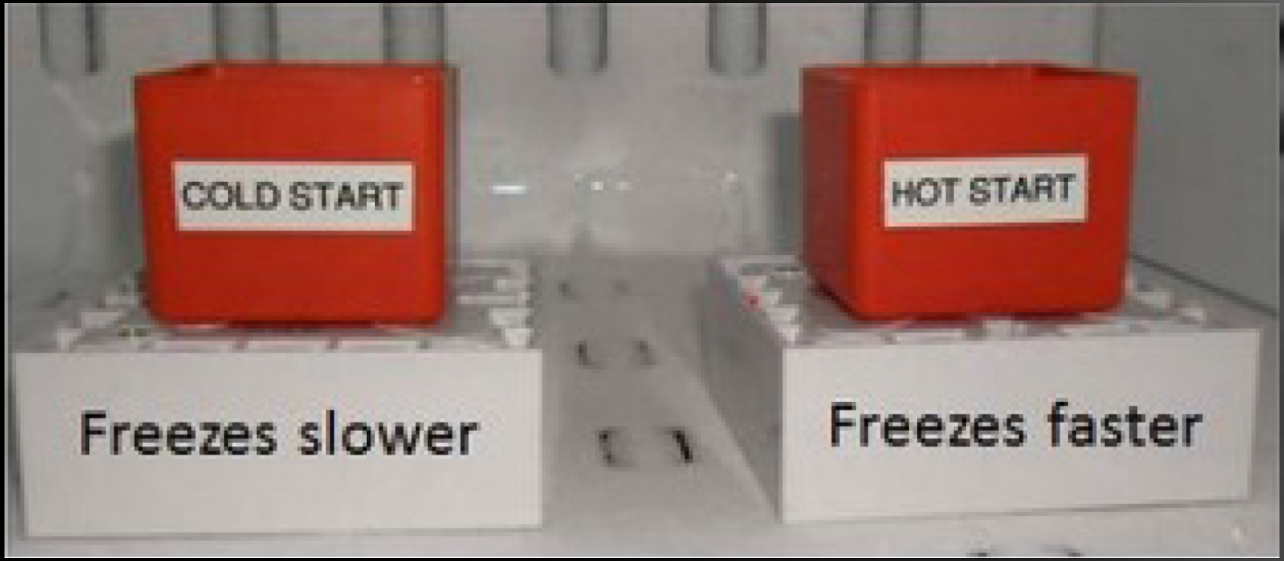
What is the Mpemba effect?

"Hot water <u>can</u> freeze faster than cold water"



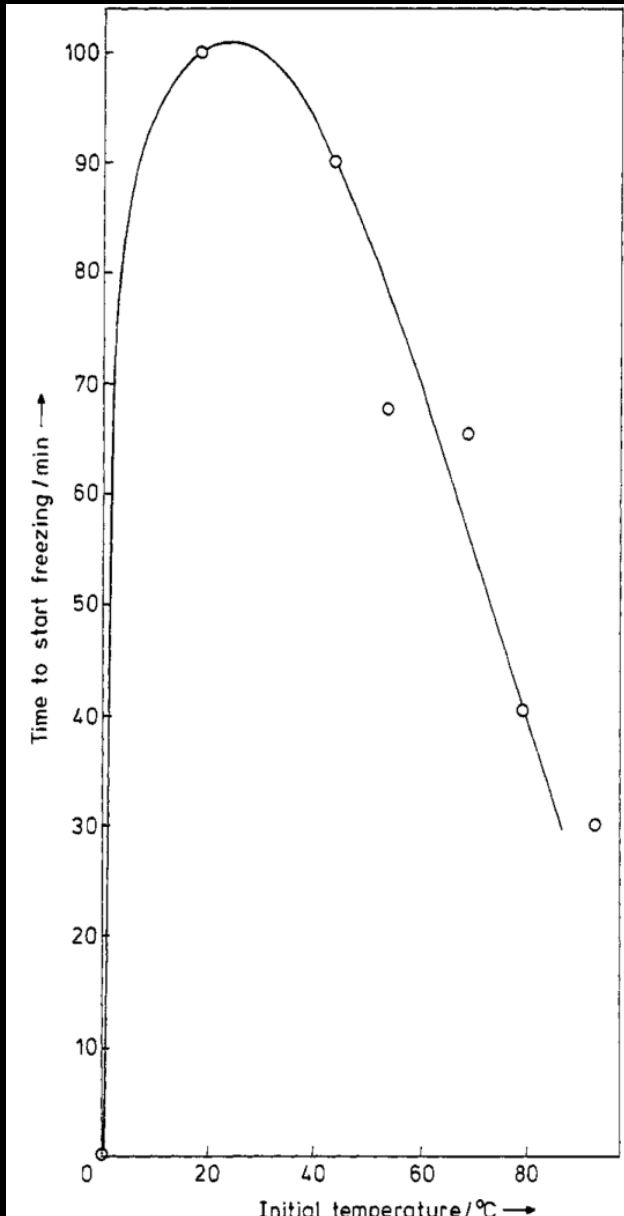
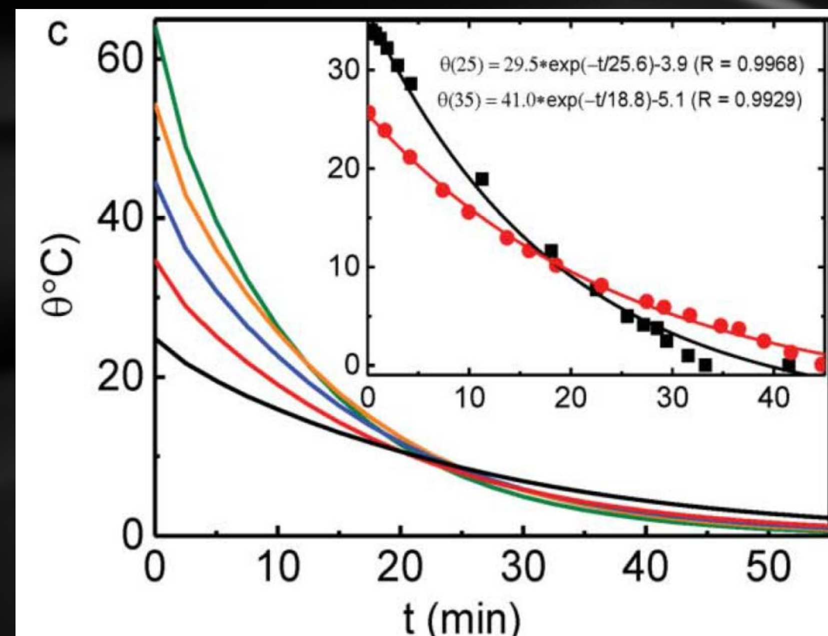


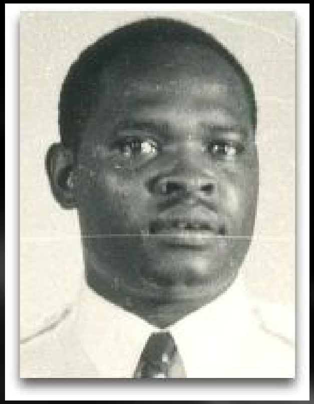
Figure 1 Plot of time for water to start freezing against initial temperature of water

Mpemba and Osborne,
Phys. Educ. **4**, 172 (1969)

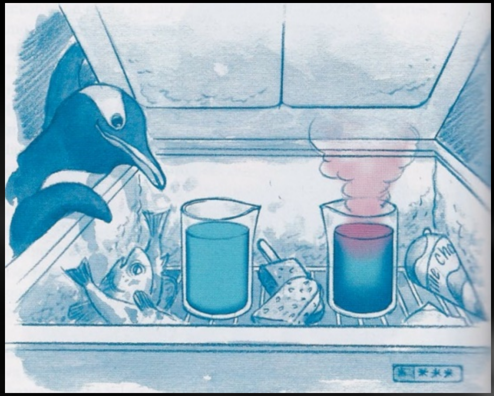
C. Q. Sun, Temperature **2**, 38 (2015)



Why the name?



- In 1963, Erasto B. Mpemba (b. 1950, Tanzania) accidentally noticed that using boiled milk to make ice cream required less time than using cold milk.
- His physics teachers in secondary and high schools told him that he was confused.
- But he kept observing the same paradoxical results in his private experiments.



(Credit: Bruno Vacaro)

- Dr. D. G. Osborne (University College in Dar es Salaam) was invited to give a talk in Mpemba’s high school.
- Young Mpemba asked the same question to Dr. Osborne.
- Dr. Osborne promised he would experimentally test the claim and encouraged Mpemba to repeat the experiment himself.

- In 1969, Mpemba and Osborne reported experimental results showing this counterintuitive effect.

Mpemba and Osborne, Phys. Educ. **4**, 172 (1969)

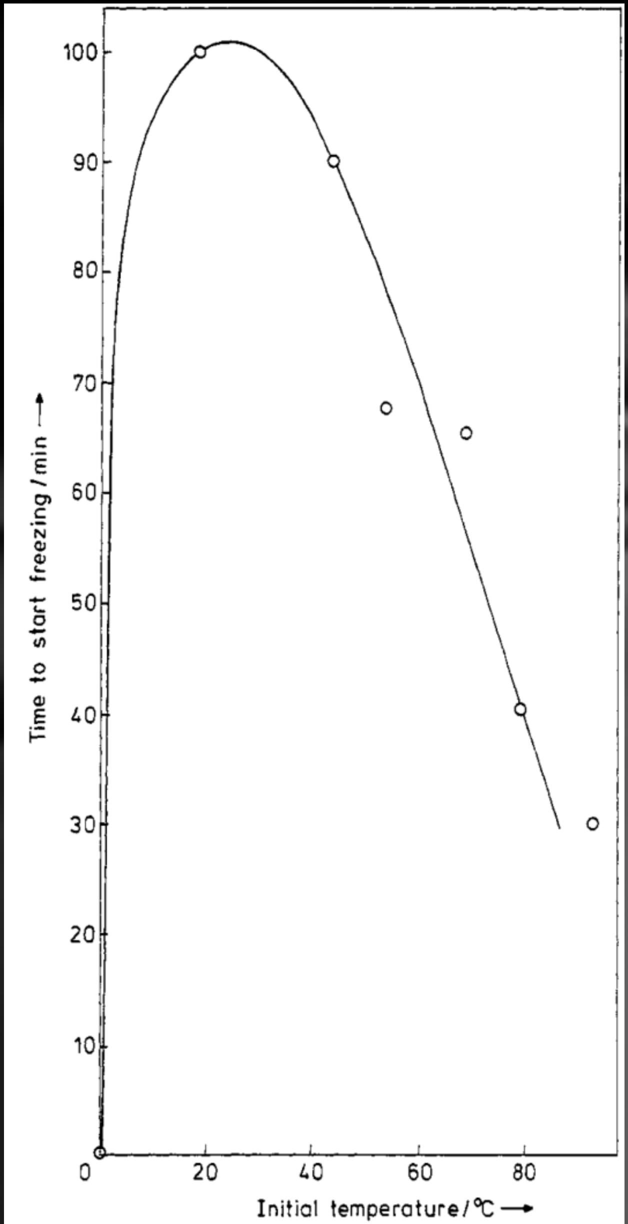
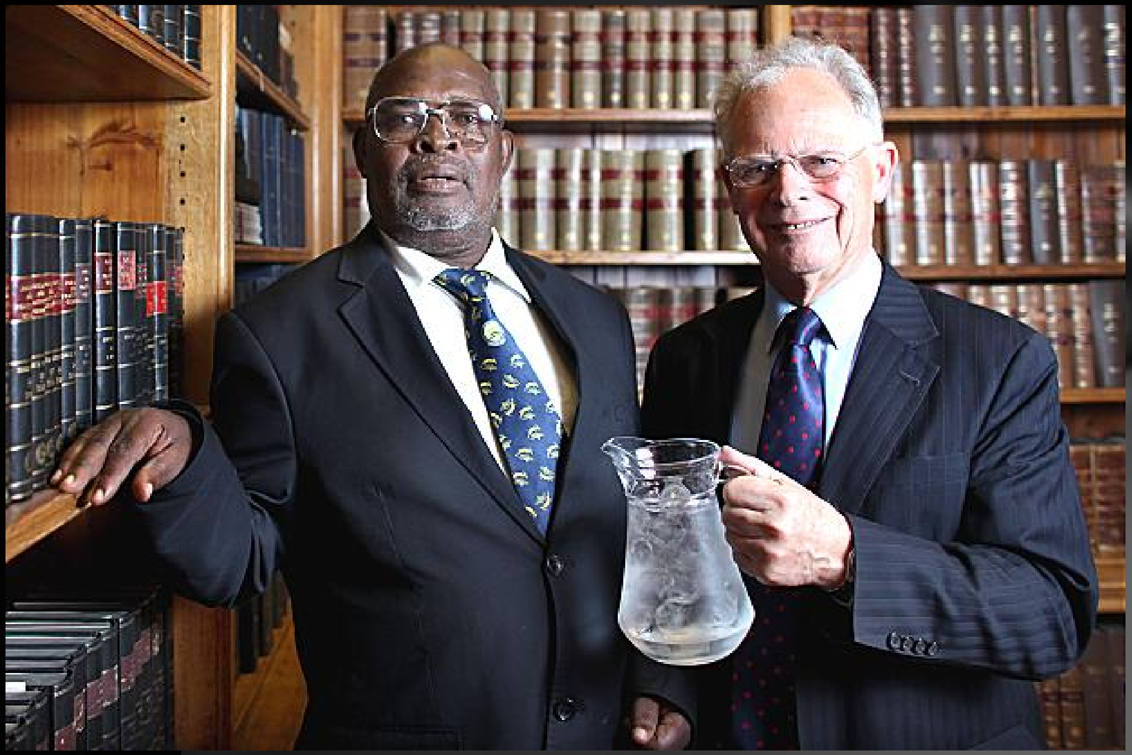


Figure 1 Plot of time for water to start freezing against initial temperature of water

Erasto B. Mpemba talking at the TEDxDar event (Dar es Salaam, November 2011)

Erasto B. Mpemba and Denis G. Osborne in London (2013).



Mpemba became Principal Game Officer in the Ministry of Natural Resources and Tourism in the Wildlife Division (Tanzania). He is now retired.

The problem had been around for millennia, with philosophers such as Aristotle, R. Bacon, G. Marliani, F. Bacon, and Descartes pondering over it.

The fact that the water has previously been warmed contributes to its freezing quickly; for so it cools sooner

Aristotle
(384–322 BC)

If cold water and hot water are poured on a cold place, as upon ice, the hot water freezes more quickly

Roger Bacon
(1214–1294)

Water slightly warm is more easily frozen than quite cold

Francis Bacon
(1561–1626)

Experience shows that water which has been kept for a long time on the fire freezes sooner than other water

René Descartes
(1596–1650)

Analogous effects have been part of the popular belief in cold countries (like Canada).

(Credit: Bruno Vacaro)

- Scientists have suggested a number of theories (evaporation, dissolved gases, convection, supercooling, bonding of water molecules, …).
- No full consensus on whether or not the effect might be an artifact of the experimental procedures.

Questioning the Mpemba effect: hot water does not cool more quickly than cold

Henry C. Burridge[1,2] & Paul F. Linden[1]

The Mpemba effect is the name given to the assertion that it is quicker to cool water to a given temperature when the initial temperature is higher. This assertion seems counter-intuitive and yet references to the effect go back at least to the writings of Aristotle. Indeed, at first thought one might consider the effect to breach fundamental thermodynamic laws, but we show that this is not the case. We go on to examine the available evidence for the Mpemba effect and carry out our own experiments by cooling water in carefully controlled conditions. We conclude, somewhat sadly, that there is no evidence to support meaningful observations of the Mpemba effect.

Sci. Rep. **6**, 37665 (2016).

The role of additional parameters

Newton's law of cooling: $\dot{T} = -\lambda(T - T_s)$

$\Rightarrow$ NO Mpemba effect

$$\text{Mpemba effect} \Rightarrow \begin{cases} \dot{T} & = F_T(T, \{X_j\}) \\ \dot{X}_i & = F_i(T, \{X_j\}) \end{cases}$$

Paradigmatic example:

A granular gas!

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Nonequilibrium thermodynamics of the Markovian Mpemba effect and its inverse

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Edited by David A. Weitz, Harvard University, Cambridge, MA, and approved April 4, 2017 (received for review January 23, 2017)

Under certain conditions, it takes a shorter time to cool a hot system than to cool the same system initiated at a lower temperature. This phenomenon—the "Mpemba effect"—was first observed in water and has recently been reported in other systems. Whereas several detail-dependent explanations were suggested for some of these observations, no common underlying mechanism is known. Using the theoretical framework of

water and clathrate hydrates, they are all substance specific and thus cannot explain the Mpemba effect observed in other substances, e.g., in magneto-resistance alloys or granular systems.

In this paper, we consider anomalous cooling processes in the general framework of nonequilibrium statistical mechanics. For systems undergoing Markovian dynamics, we provide a sufficient condition, accompanied with heuristic intuition for its appear-

the cold system; i.e., the Mpemba effect cannot occur. On the other hand, this effect has been observed in water (4) and more recently in several other substances, e.g., nanotube resonators (5), magneto-resistance alloys (6), clathrate hydrates (7), and granular systems (8).

8. Lasanta A, Vega Reyes F, Prados A, Santos A (2016) When the hotter cools more quickly: Mpemba effect in granular fluids. *arXiv*:1611.04948.

Outline

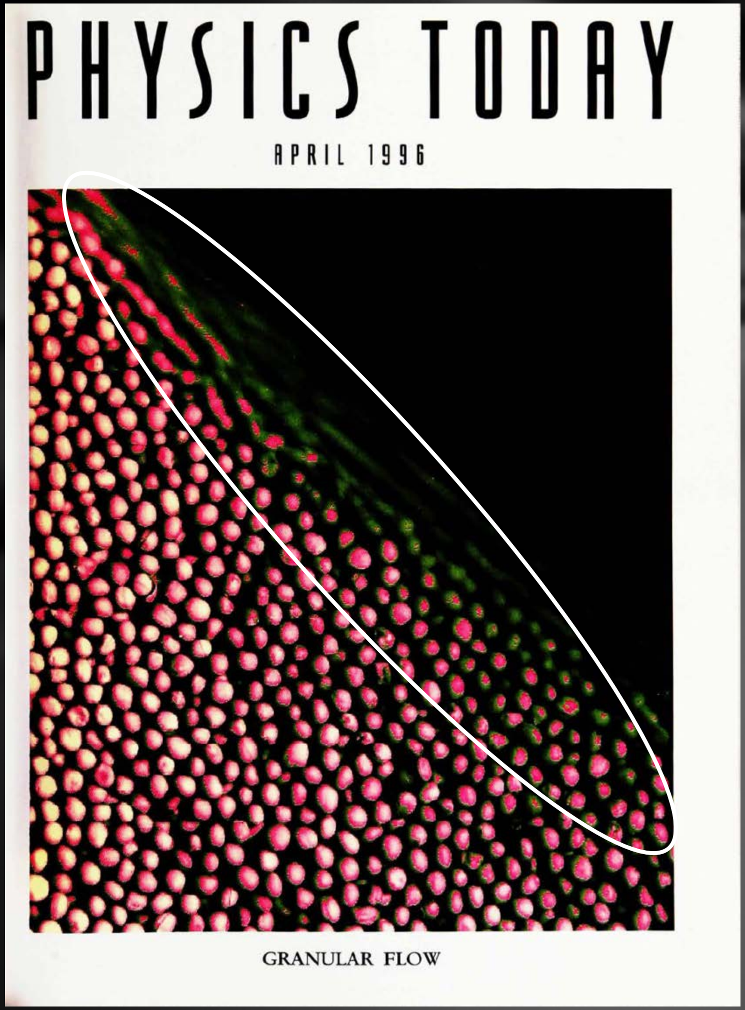
- What is the Mpemba effect?
- What is a granular gas?
- Mpemba effect in granular gases
- Mpemba effect in molecular gases

What is a granular *gas*?

- When the granular matter is driven and energy is fed into the system (e.g., by shaking), the granular material is said to *fluidize*.

- **Granular gas**: Mean free path much larger than the grain size

KINETIC THEORY



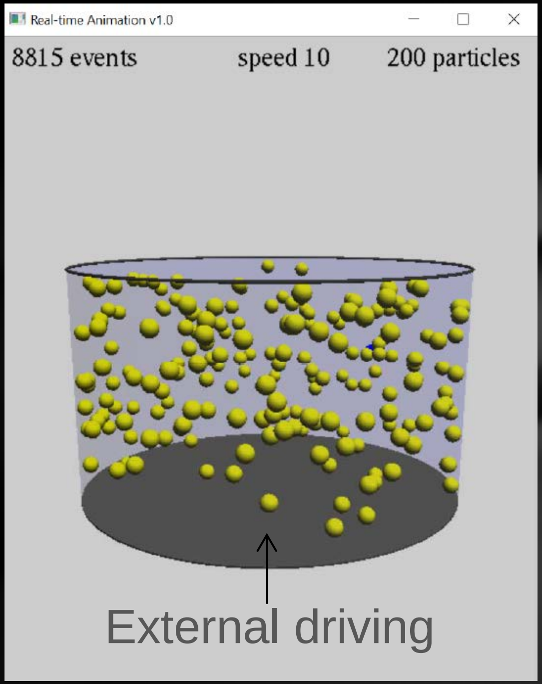
Granular gas: Dissipative collisions

Temperature:

$$T = \frac{m}{3}\langle v^2 \rangle$$

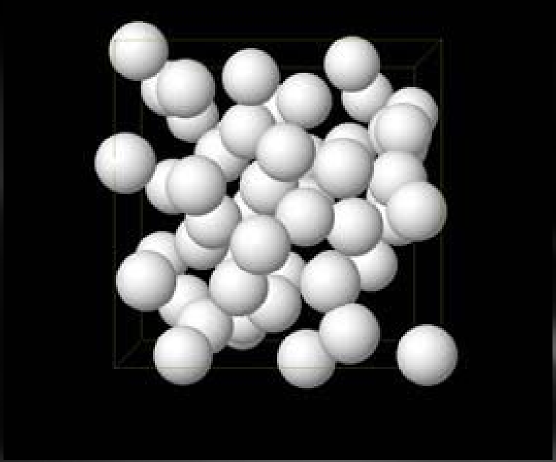
Excess kurtosis:

$$a_2 = \frac{3}{5}\frac{\langle v^4 \rangle}{\langle v^2 \rangle^2} - 1$$

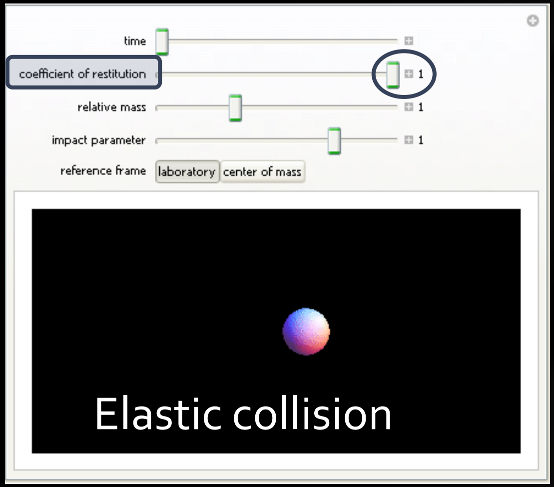


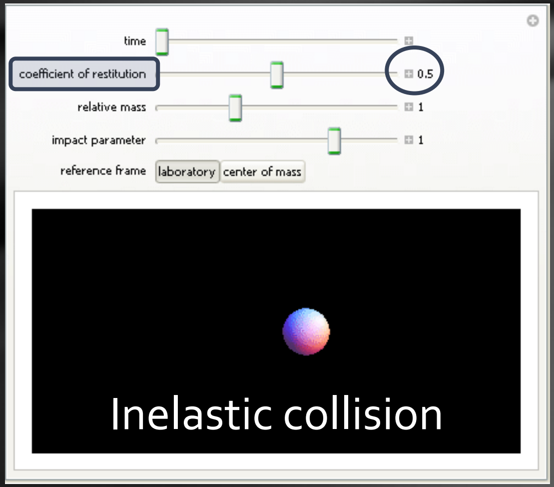
Demo by Sergei Mechov

Standard model of a granular gas:
A gas of identical *inelastic smooth* hard spheres



Constant coefficient of *normal* restitution α





http://demonstrations.wolfram.com/InelasticCollisionsOfTwoSpheres/

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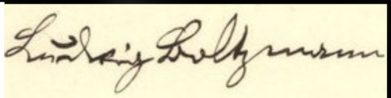
Our approach: Kinetic Theory [arXiv:1611.04948]



(1844-1906)

(Cartoon by Bernhard Reischl, University of Vienna)

Homogeneous Boltzmann equation:

$$\partial_t f(\mathbf{v},t) \underbrace{- \frac{\xi^2}{2}\left(\frac{\partial}{\partial \mathbf{v}}\right)^2 f(\mathbf{v},t)}_{\text{External driving}} = \underbrace{J[\mathbf{v},t|f]}_{\text{Inelastic collisions}}$$

$$\text{“Newton-like” cooling equation: } \dot{T} = -\frac{2\kappa}{3}\left(\mu_2 T^{3/2} - \mu_{2,s} T_s^{3/2}\right)$$

$$= F_T(T|f)$$

Collisional moments

$$\text{Equation for the kurtosis: } \dot{a}_2 = F_{a_2}(T, a_2|f)$$

Approximations

1. $|a_2| \ll 1, \quad a_3, a_4, \ldots$ negligible $\Rightarrow \underbrace{\begin{cases} \dot{T} & = F_T(T|f) \to F_T(T, a_2) \\ \dot{a}_2 & = F_{a_2}(T, a_2|f) \to F_{a_2}(T, a_2) \end{cases}}_{\text{Closed set}}$

2. $\theta \equiv \dfrac{T}{T_s} \sim 1$

$$\theta(\tau) = 1 + \frac{1}{\gamma}\left[(\lambda_+ - \mu_{2,s})(\theta_0 - 1) - \frac{2}{3}\mu_2^{(1)}(a_{2,0} - a_{2,s})\right] e^{-\lambda_- \tau}$$
$$- \frac{1}{\gamma}\left[(\lambda_- - \mu_{2,s})(\theta_0 - 1) - \frac{2}{3}\mu_2^{(1)}(a_{2,0} - a_{2,s})\right] e^{-\lambda_+ \tau}$$

Phase diagram for the Mpemba effect

Initial condition A:

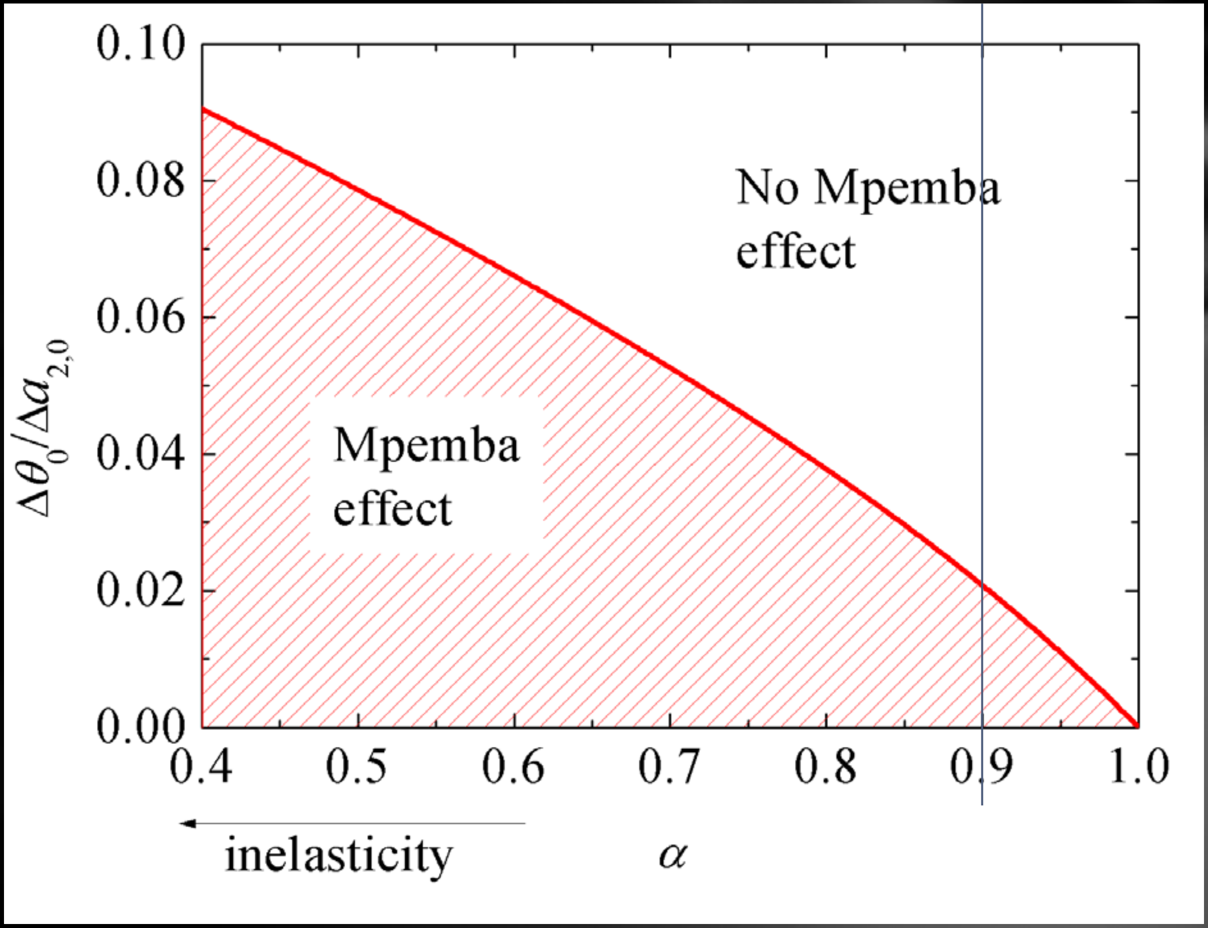
$$\theta_0 = \theta_A, a_{2,0} = a_{2A}$$

Initial condition B:

$$\theta_0 = \theta_B, a_{2,0} = a_{2B}$$

$$\Delta\theta_0 \equiv \theta_A - \theta_B$$

$$\Delta a_{2,0} \equiv a_{2A} - a_{2B}$$



Crossover time

Initial condition A:

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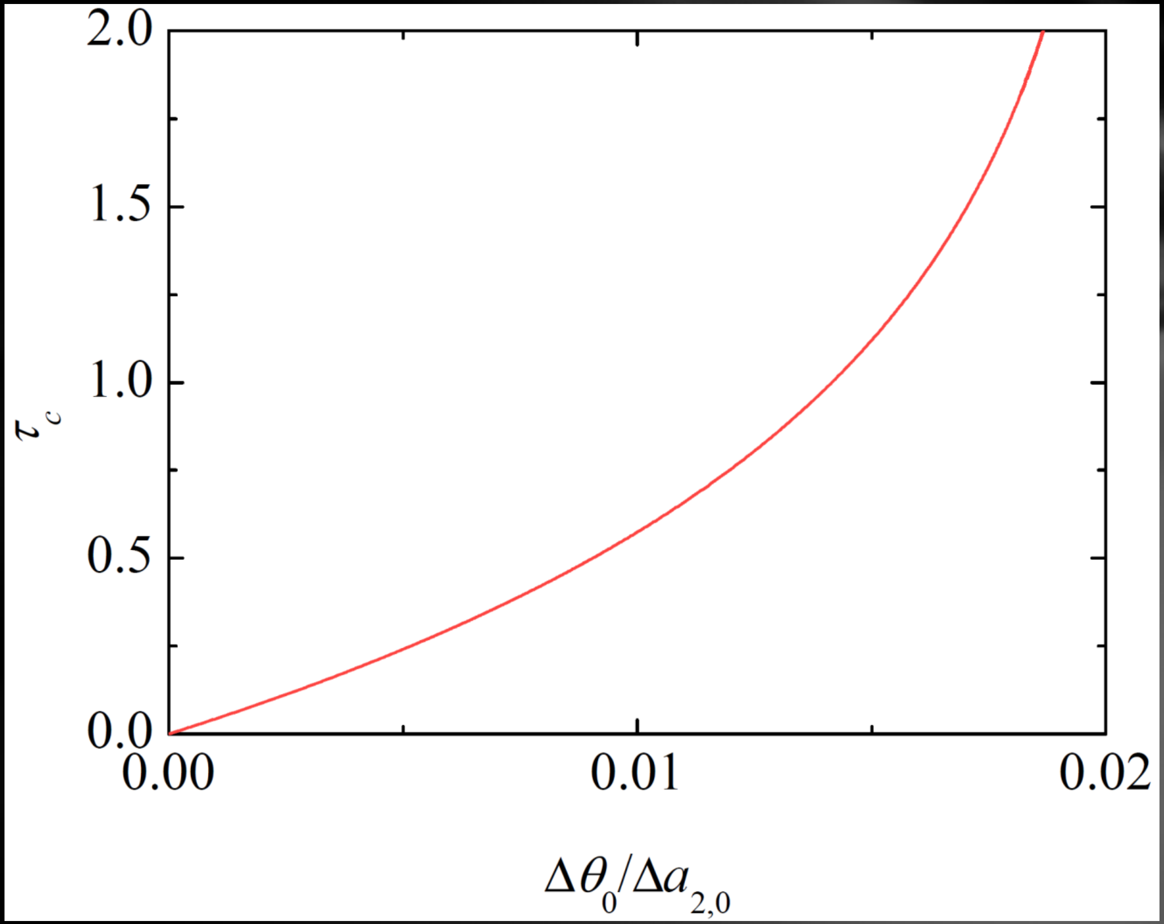
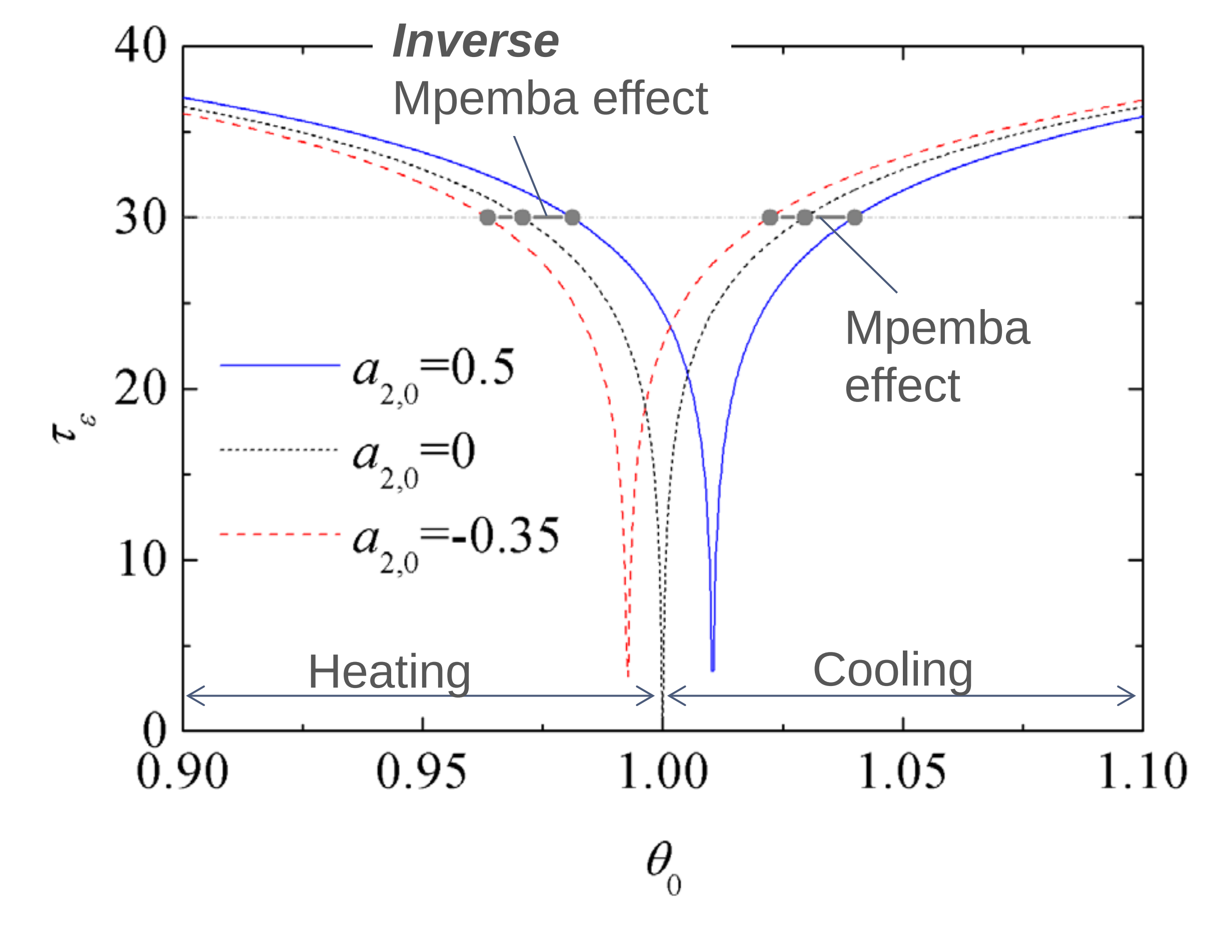
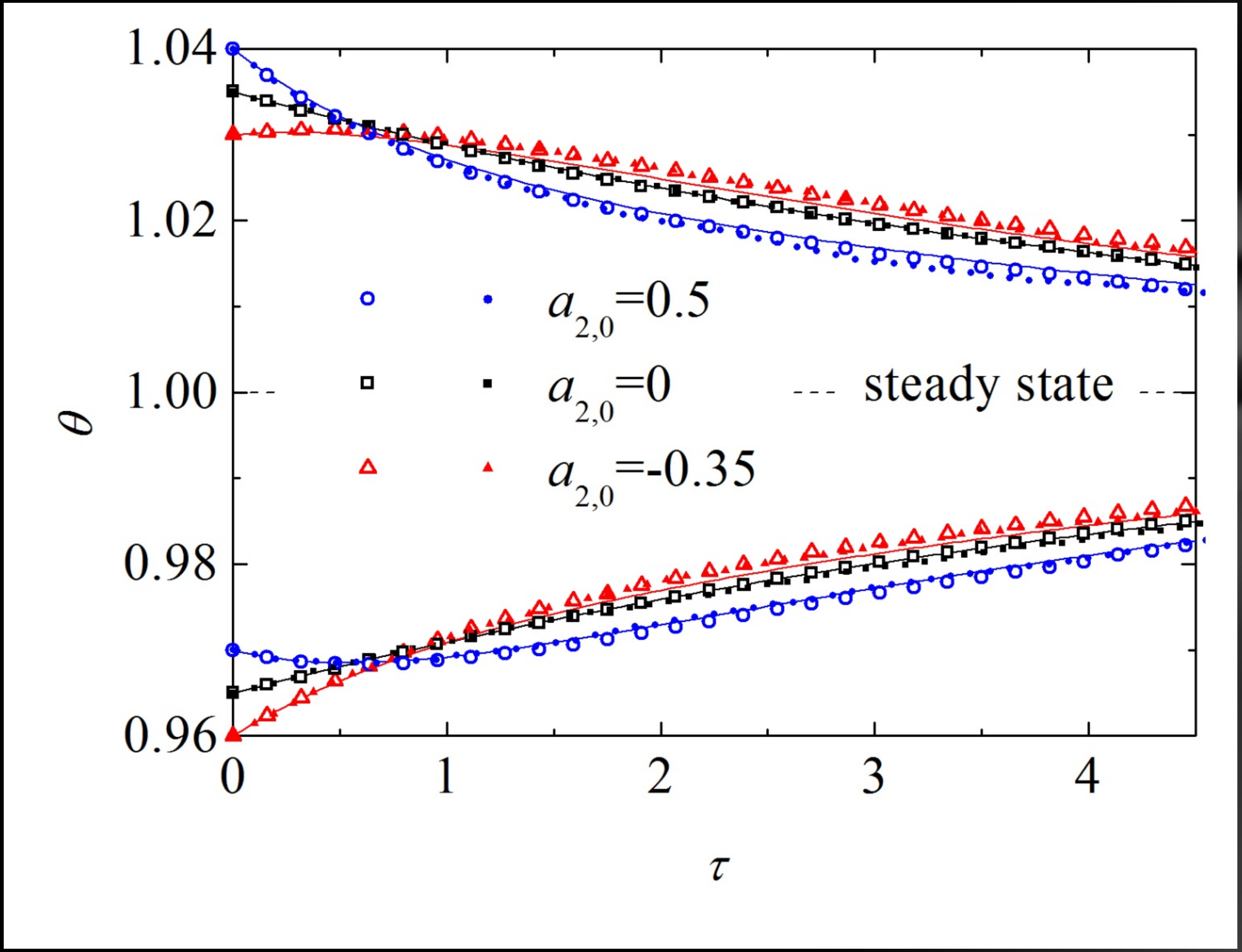


Relaxation time to the steady state



Comparison with computer simulations (DSMC & MD)



Mpemba effect

Inverse
Mpemba effect

Is the effect limited to $T_0 \sim T_s$?

"Newton-like" cooling equation: $\dot{T} = -\frac{2\kappa}{3}\left(\cancel{\mu_2 T^{3/2}} - \cancel{\mu_{2,s} T_s^{3/2}}\right)$

1. $T_0 \ll T_s \Rightarrow \dot{T} \approx \frac{2\kappa}{3}\mu_{2,s}T_s^{3/2} \Rightarrow$ NO (inverse) Mpemba effect

2. $T_0 \gg T_s \Rightarrow \dot{T} \approx -\frac{2\kappa}{3}\mu_2 T^{3/2} \Rightarrow$ Possibility of Mpemba effect (homogeneous cooling state)

Phase diagram if $T_0 >> T_s$

Initial condition A:

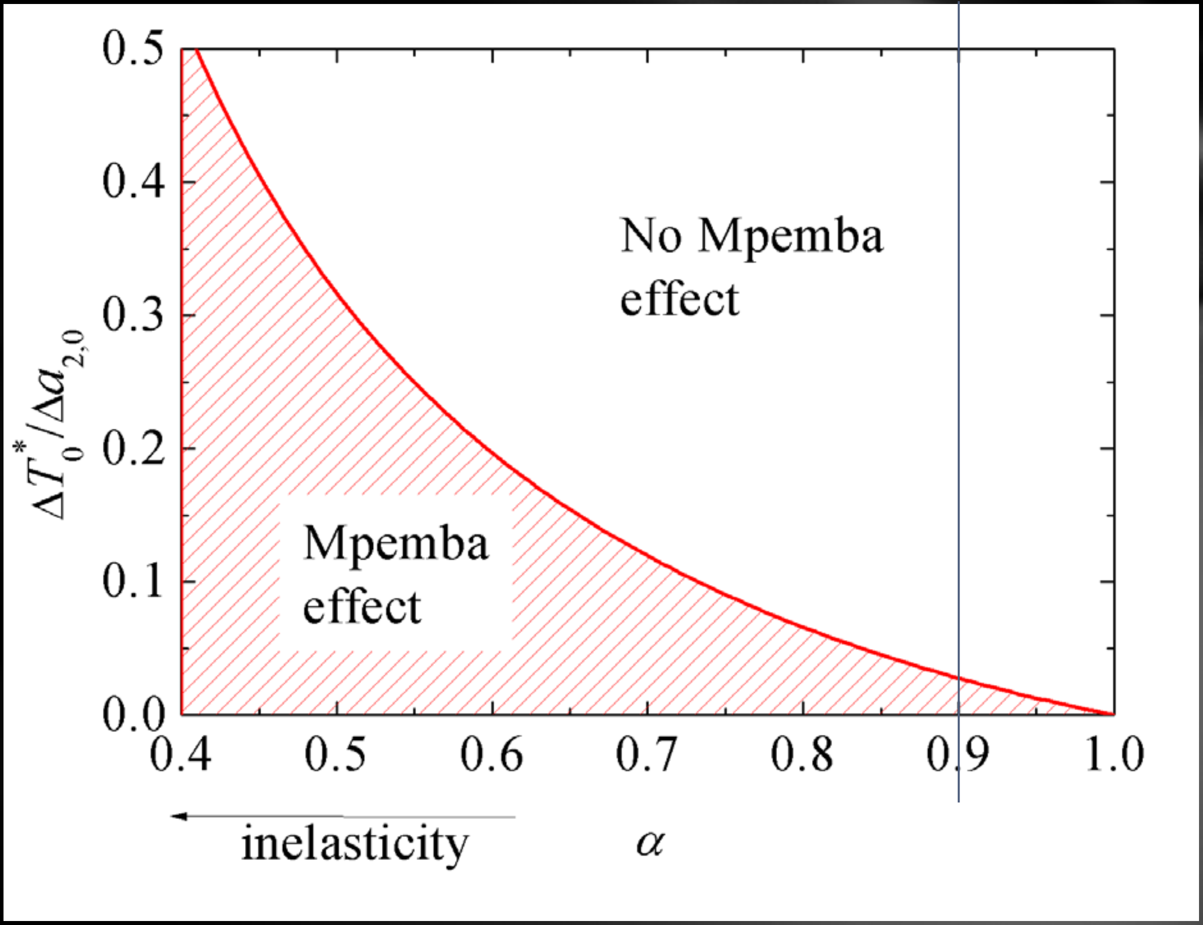
$$T_0 = T_A, a_{2,0} = a_{2A}$$

Initial condition B:

$$T_0 = T_B, a_{2,0} = a_{2B}$$

$$\Delta T_0^* \equiv \frac{T_A - T_B}{T_A}$$

$$\Delta a_{2,0} \equiv a_{2A} - a_{2B}$$



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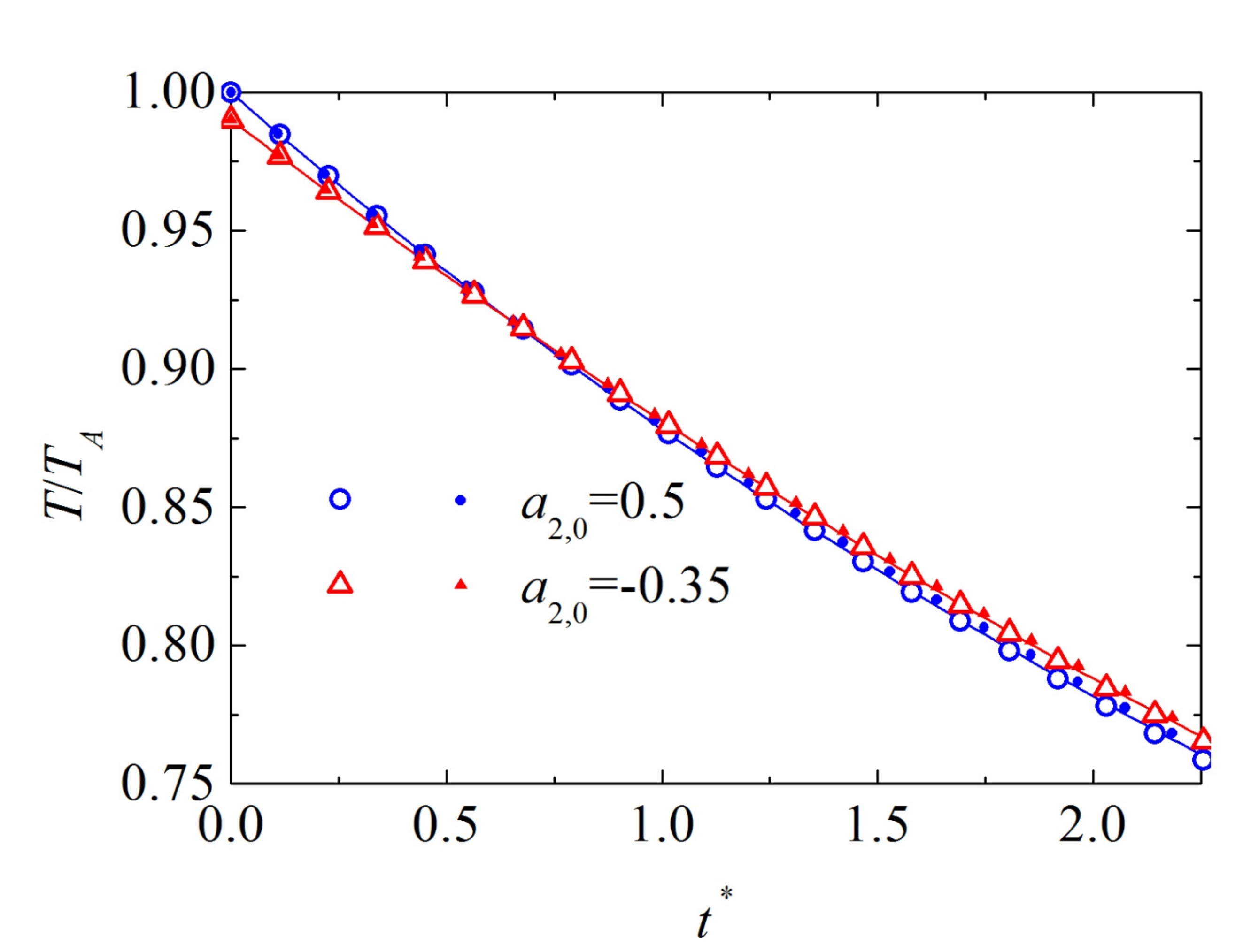


Generalized phase diagram

Initial condition A:

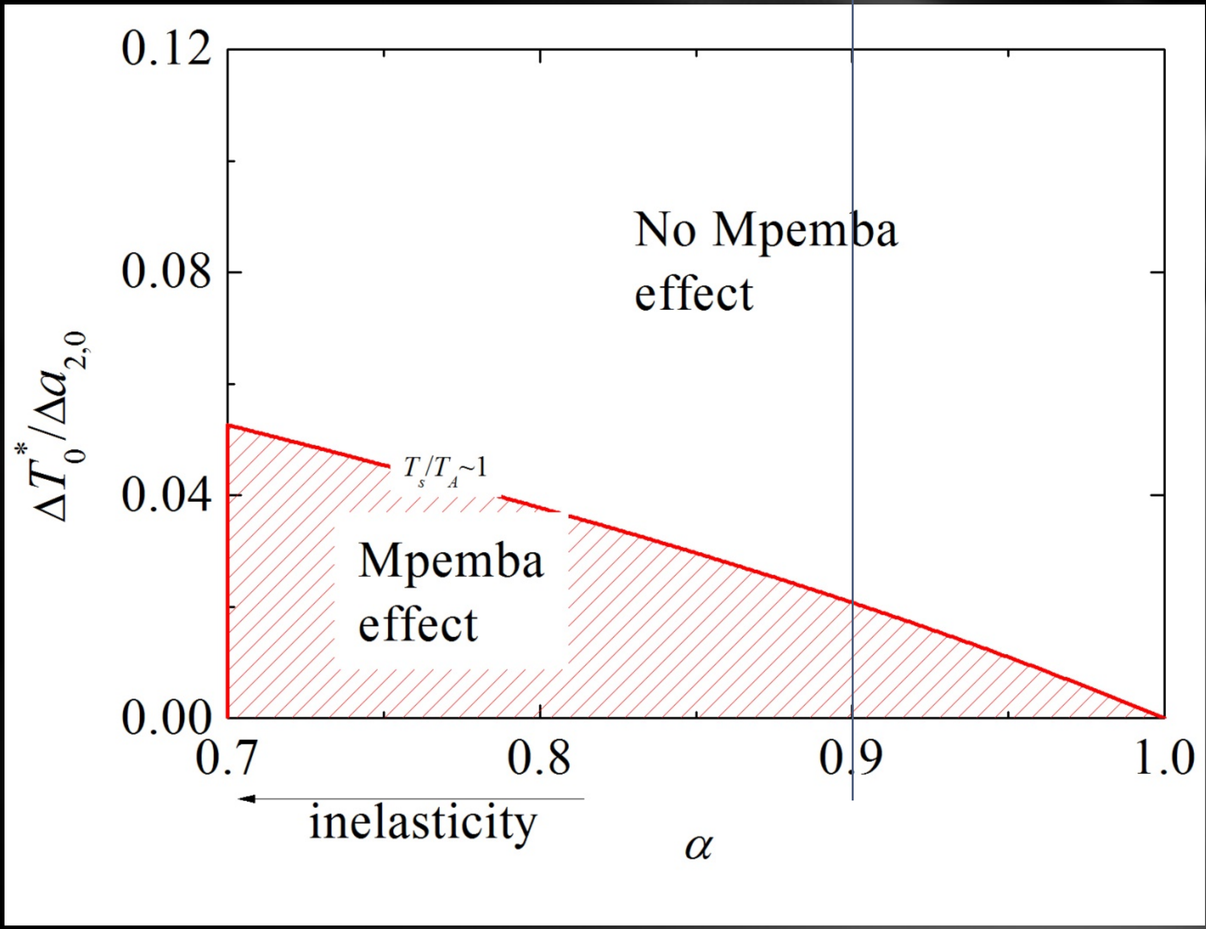
$$T_0 = T_A, a_{2,0} = a_{2A}$$

Initial condition B:

$$T_0 = T_B, a_{2,0} = a_{2B}$$

$$\Delta T_0^* \equiv \frac{T_A - T_B}{T_A}$$

$$\Delta a_{2,0} \equiv a_{2A} - a_{2B}$$



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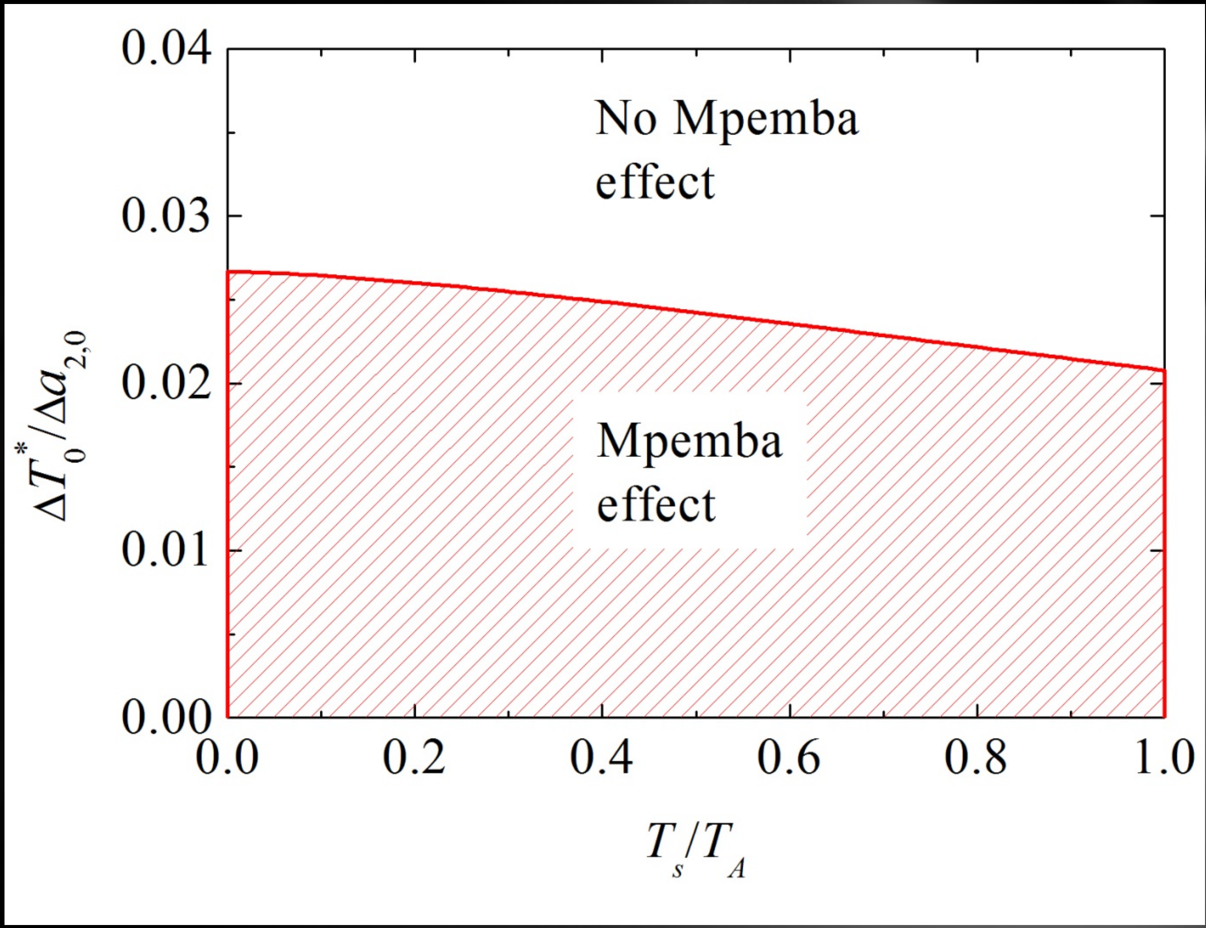

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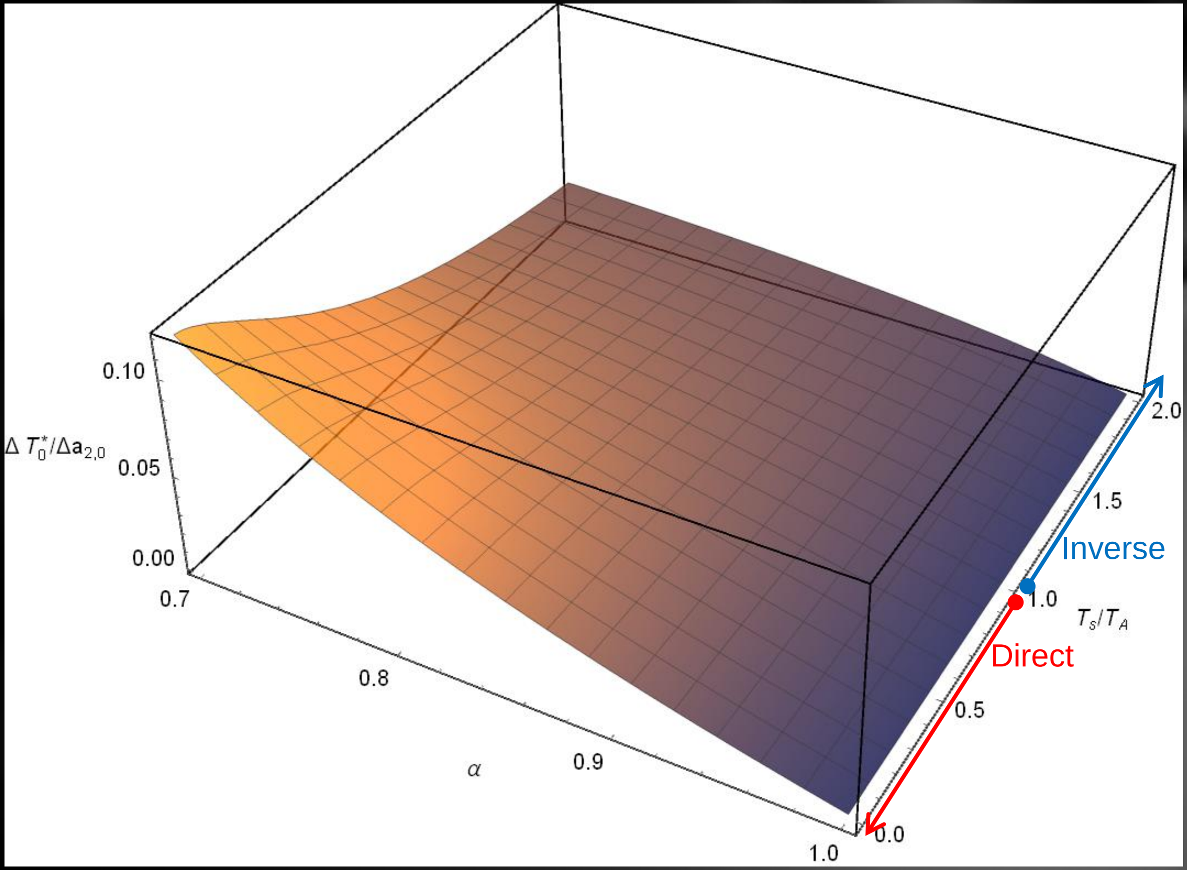

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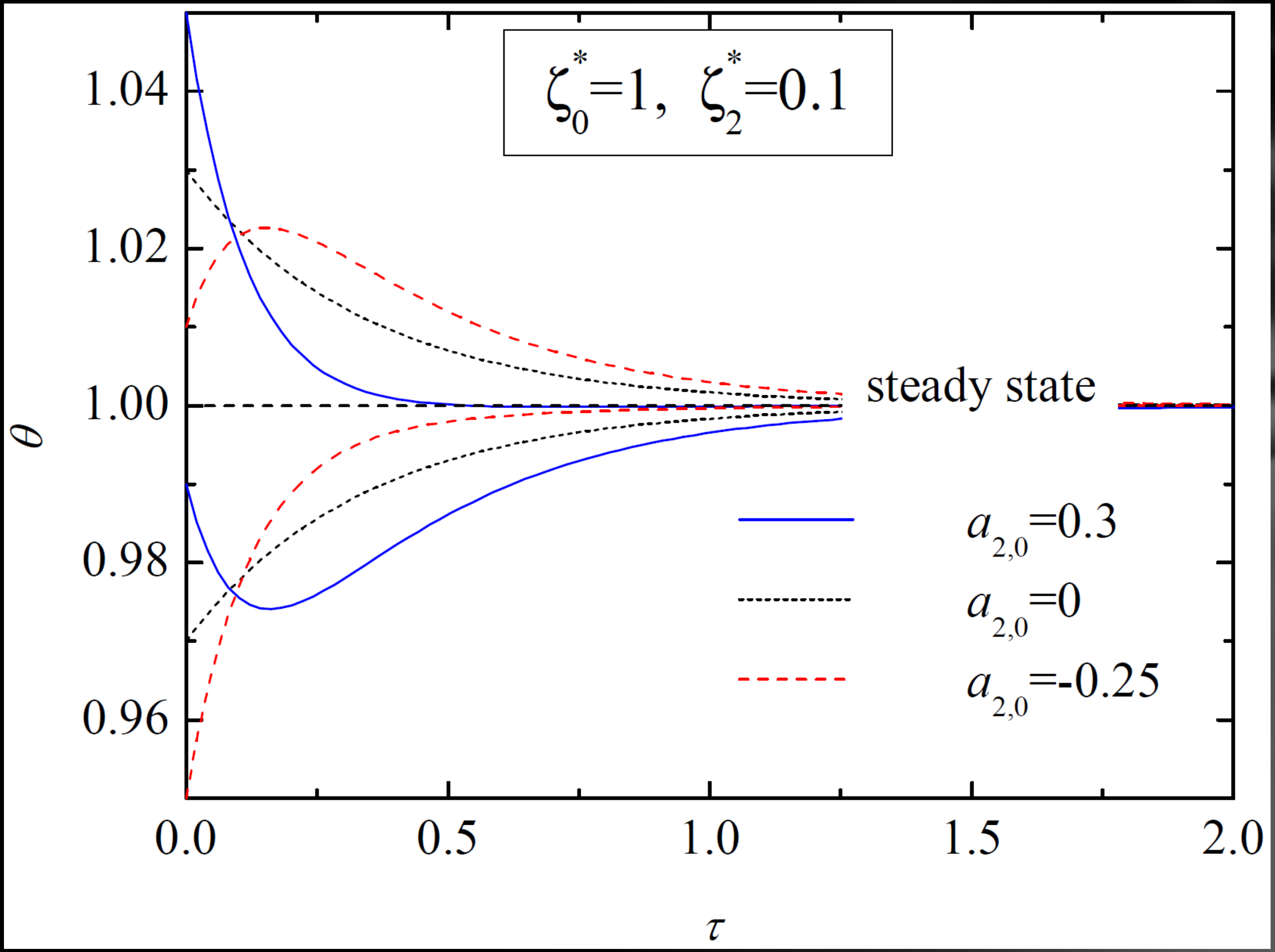
Mpemba effect in *molecular* gases

Molecular gas subject to a *nonlinear* drag

(ongoing project)

$$\partial_t f(\mathbf{v},t) \underbrace{- \frac{\partial}{\partial \mathbf{v}} \cdot \left[\zeta(v) \left(\mathbf{v} + \frac{T_s}{m} \frac{\partial}{\partial \mathbf{v}} \right) f(\mathbf{v},t) \right]}_{\text{Drag term}} = \underbrace{J[\mathbf{v},t|f]}_{\text{Elastic collisions}}$$

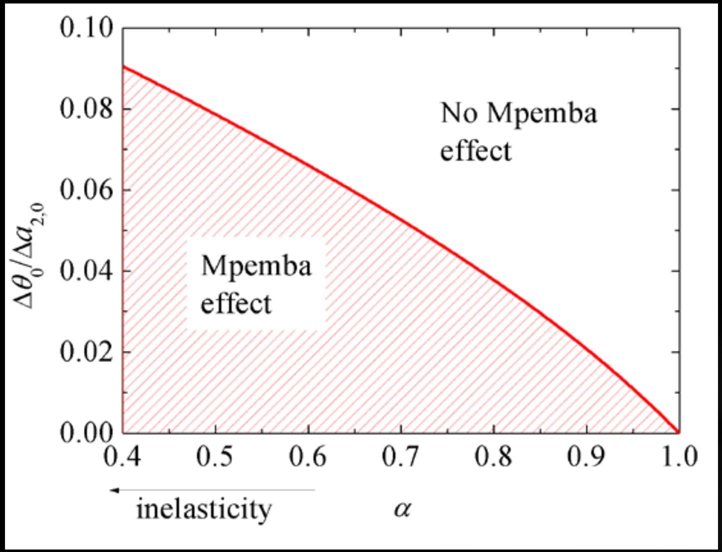
Nonlinear drag: $\zeta(v) = \zeta_0 + \zeta_2 v^2$

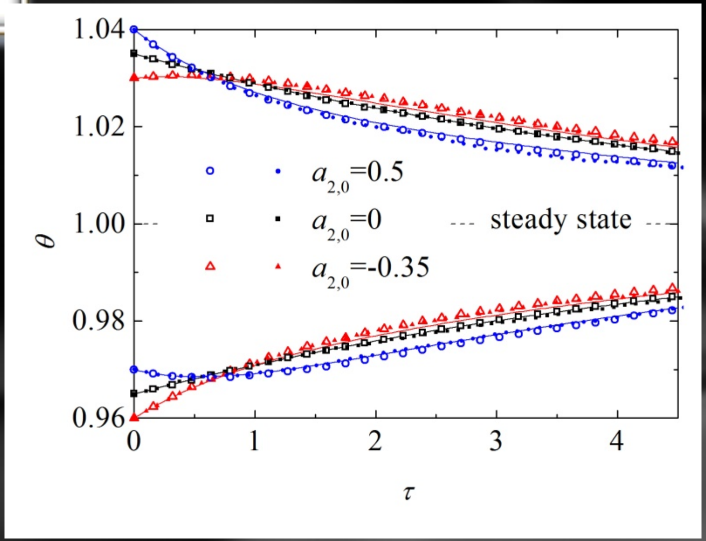


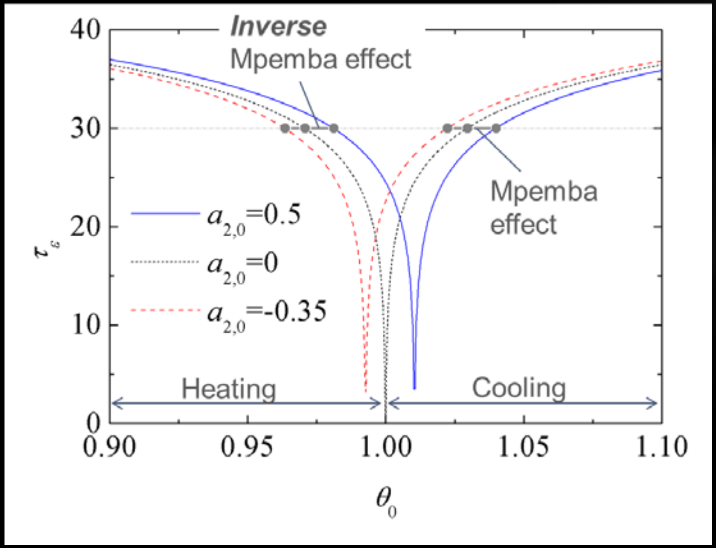
$\zeta_0^*=1,\ \zeta_2^*=0.1$
1.04
1.02
1.00
0.98
0.96
θ
steady state
$a_{2,0}=0.3$
$a_{2,0}=0$
$a_{2,0}=-0.25$
0.0
0.5
1.0
1.5
2.0
τ

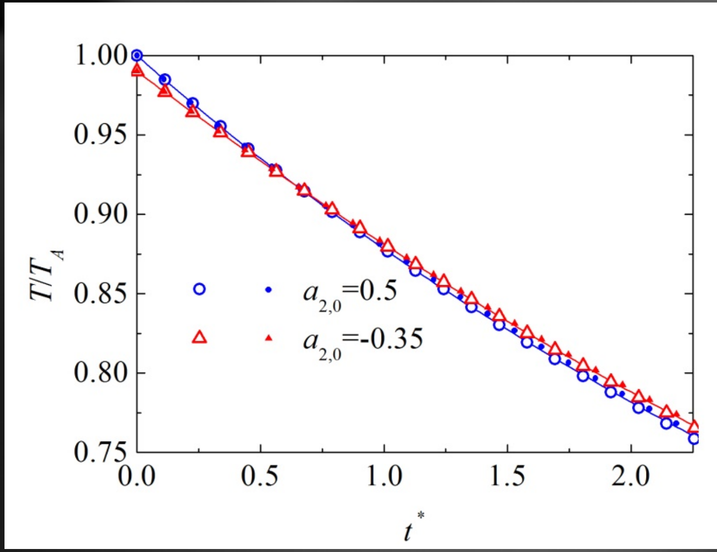
- For a given system, the Mpemba effect can be expected if $dT/dt=F_T(T,\{X_j\})$.
- In a homogeneous granular gas, the simplest approach [$dT/dt=F_T(T,a_2)$, $da_2/dt=F_a(T,a_2)$] describes the effect very accurately.
- The effect also exists in a molecular gas (elastic collisions) driven by a *nonlinear* drag (ongoing work).


THANK
Y U!


No Mpemba effect
Mpemba effect
inelasticity
α
$\Delta\theta_0/\Delta a_{2,0}$
0.10
0.08
0.06
0.04
0.02
0.00
0.4 0.5 0.6 0.7 0.8 0.9 1.0


$a_{2,0}$=0.5
$a_{2,0}$=0
$a_{2,0}$=-0.35
steady state
θ
τ
1.04
1.02
1.00
0.98
0.96
0 1 2 3 4


Inverse
Mpemba effect
Mpemba effect
$a_{2,0}$=0.5
$a_{2,0}$=0
$a_{2,0}$=-0.35
Heating
Cooling
τ_ε
θ_0
40
30
20
10
0
0.90 0.95 1.00 1.05 1.10


$a_{2,0}$=0.5
$a_{2,0}$=-0.35
T/T_A
t^*
1.00
0.95
0.90
0.85
0.80
0.75
0.0 0.5 1.0 1.5 2.0