Mpemba effect in granular media

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What is the Mpemba effect?

"Hot water can freeze faster than cold water"

(Credit: Leif Parsons)

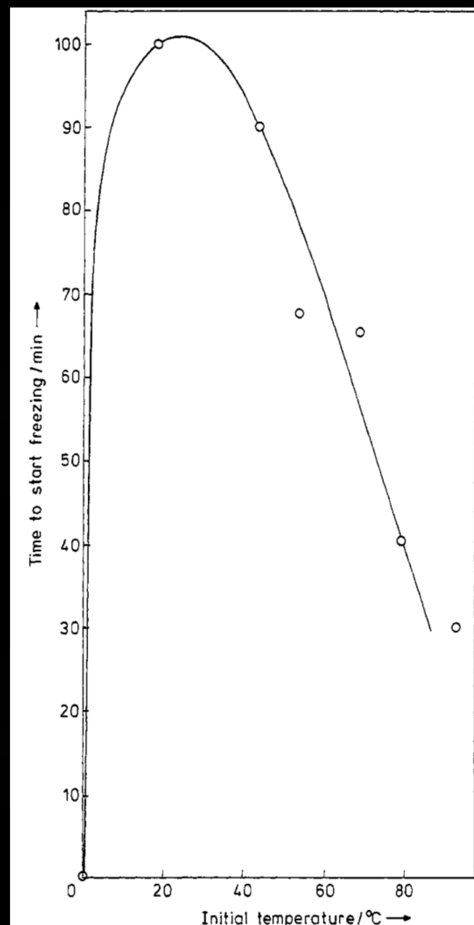
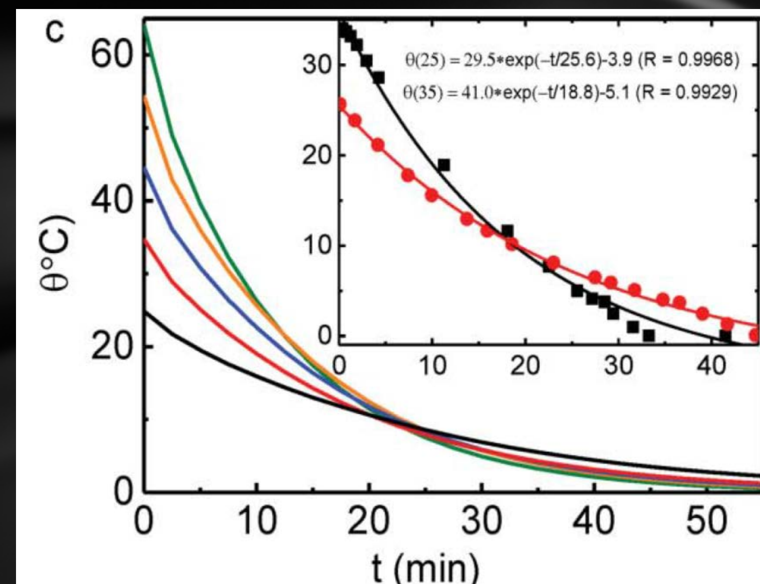


Figure 1 Plot of time for water to start freezing against initial temperature of water

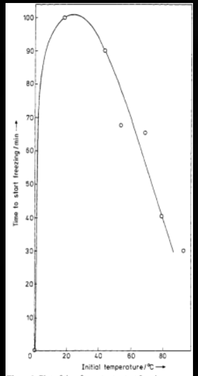
Mpemba and Osborne, Phys. Educ. **4**, 172 (1969)

C. Q. Sun, Temperature **2**, 38 (2015)



Why the name?

Erasto B. Mpemba (left) and Denis G. Osborne (right) in London (2013).



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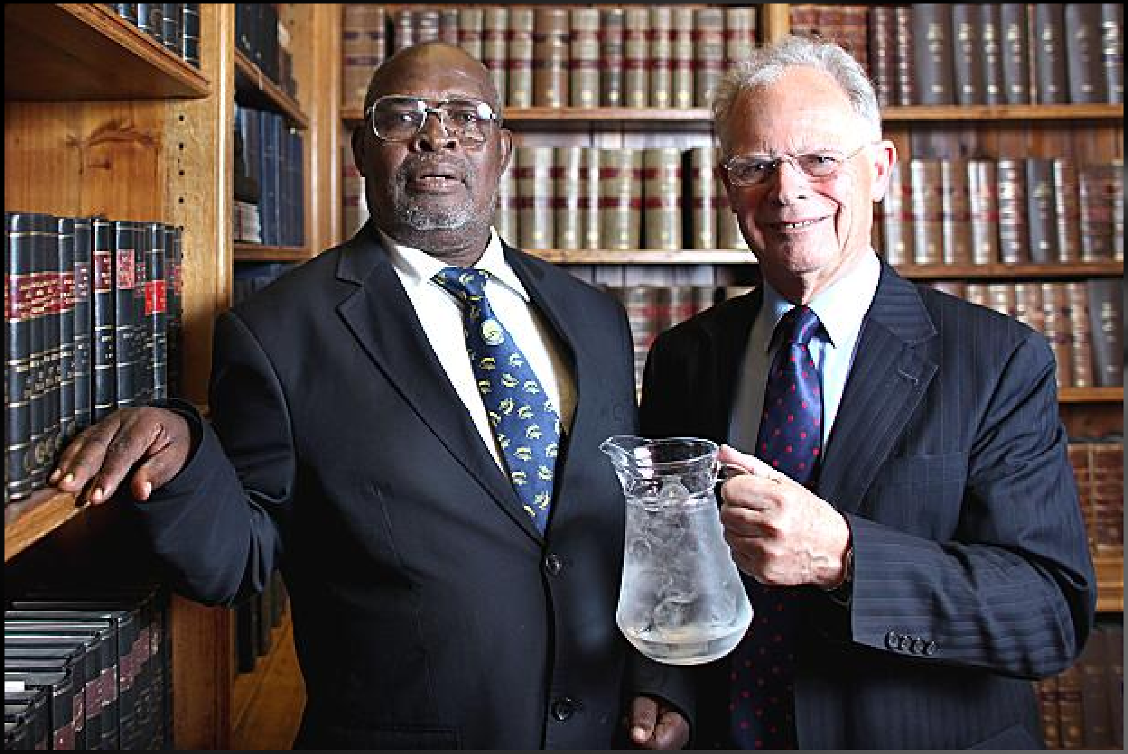


- In 1963, Erasto B. Mpemba (b. 1950), then a Tanzanian secondary school student, accidentally noticed that using boiled milk to make ice cream required less time than using cold milk.
- In 1969, Mpemba and Denis G. Osborne reported experimental results showing this paradoxical effect.

The problem had been around for millennia, with philosophers such as Aristotle, R. Bacon, F. Bacon, and Descartes pondering over it.

Aristotle
(384–322 BC)

Roger Bacon
(1214–1294)

Francis Bacon
(1561–1626)

René Descartes
(1596–1650)

- Scientists have suggested a number of theories (evaporation, dissolved gases, convection, supercooling, bonding of water molecules, …) but none have been able to satisfactorily explain this counter-intuitive physical phenomenon.
- While some scientists claim to have shown the effect using carefully controlled experiments, others deny that it exists at all and is in fact an artifact of experimental procedure.

Questioning the Mpemba effect: hot water does not cool more quickly than cold

Henry C. Burridge[1,2] & Paul F. Linden[1]

The Mpemba effect is the name given to the assertion that it is quicker to cool water to a given temperature when the initial temperature is higher. This assertion seems counter-intuitive and yet references to the effect go back at least to the writings of Aristotle. Indeed, at first thought one might consider the effect to breach fundamental thermodynamic laws, but we show that this is not the case. We go on to examine the available evidence for the Mpemba effect and carry out our own experiments by cooling water in carefully controlled conditions. We conclude, somewhat sadly, that there is no evidence to support meaningful observations of the Mpemba effect.

Sci. Rep. **6**, 37665 (2016).

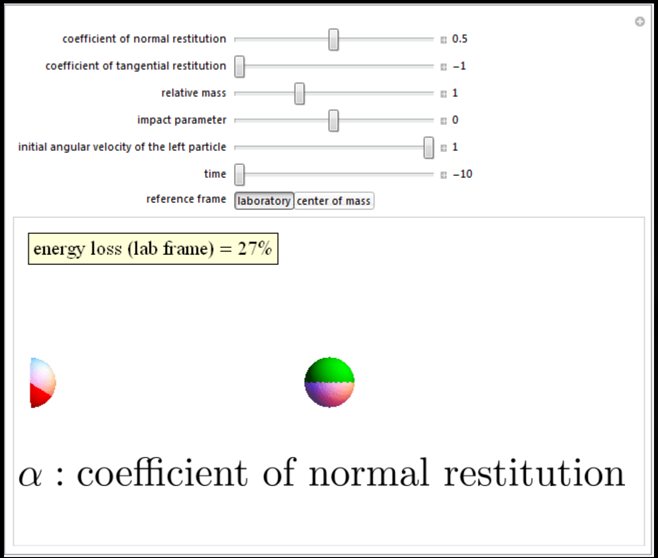
The role of additional parameters

Newton's law of cooling: $\dot{T} = -\lambda(T - T_s)$

$\Rightarrow$ NO Mpemba effect

$$\text{Mpemba effect} \Rightarrow \begin{cases} \dot{T} & = F_T(T, \{X_j\}) \\ \dot{X}_i & = F_i(T, \{X_j\}) \end{cases}$$

Our system.
Granular gas: Dissipative collisions



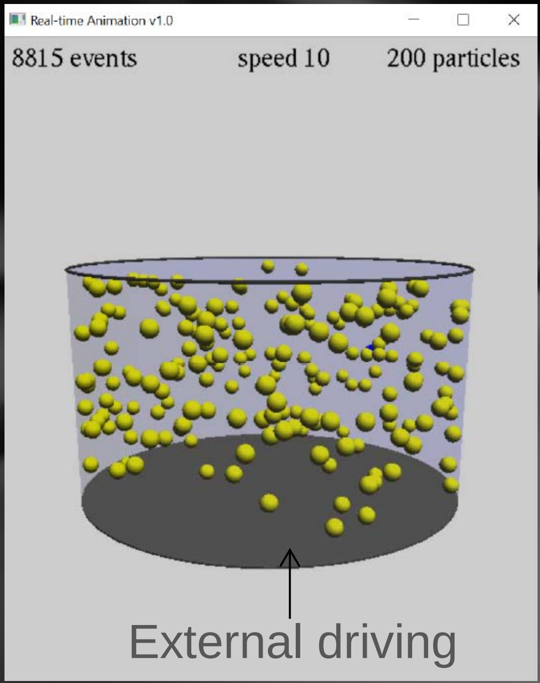
http://demonstrations.wolfram.com/InelasticCollisionsOfTwoRoughSpheres/

Temperature:

$$T = \frac{m}{3}\langle v^2 \rangle$$

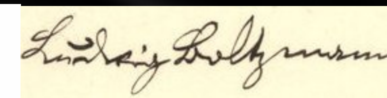
Excess kurtosis:

$$a_2 = \frac{3}{5}\frac{\langle v^4 \rangle}{\langle v^2 \rangle^2} - 1$$



Demo by Sergei Mechov

Our approach: Kinetic Theory [arXiv:1611.04948]



(1844-1906)

(Cartoon by Bernhard Reischl, University of Vienna)

Homogeneous Boltzmann equation:

$$\partial_t f(\mathbf{v},t) - \underbrace{\frac{\xi^2}{2}\left(\frac{\partial}{\partial \mathbf{v}}\right)^2 f(\mathbf{v},t)}_{\text{External driving}} = \underbrace{J[\mathbf{v},t|f]}_{\text{Inelastic collisions}}$$

"Newton-like" cooling equation:

$$\dot{T} = -\frac{2K}{3}\left(\mu_2 T^{3/2} - \mu_{2,s} T_s^{3/2}\right) = F_T(T|f)$$

Collisional moments

Equation for the kurtosis: $\dot{a}_2 = F_{a_2}(T, a_2|f)$

Approximations

1. $|a_2| \ll 1, \quad a_3, a_4, \ldots \text{ negligible} \Rightarrow \underbrace{\begin{cases} \dot{T} & = F_T(T|f) \to F_T(T, a_2) \\ \dot{a}_2 & = F_{a_2}(T, a_2|f) \to F_{a_2}(T, a_2) \end{cases}}_{\text{Closed set}}$

2. $\theta \equiv \dfrac{T}{T_s} \sim 1$

$$\theta = 1 + \frac{1}{\gamma}\left[(\lambda_+ - \mu_{2,s})(\theta_0 - 1) - \frac{2}{3}\mu_2^{(1)}(a_{2,0} - a_{2,s})\right] e^{-\lambda_- \tau}$$
$$- \frac{1}{\gamma}\left[(\lambda_- - \mu_{2,s})(\theta_0 - 1) - \frac{2}{3}\mu_2^{(1)}(a_{2,0} - a_{2,s})\right] e^{-\lambda_+ \tau}$$

Phase diagram for the Mpemba effect

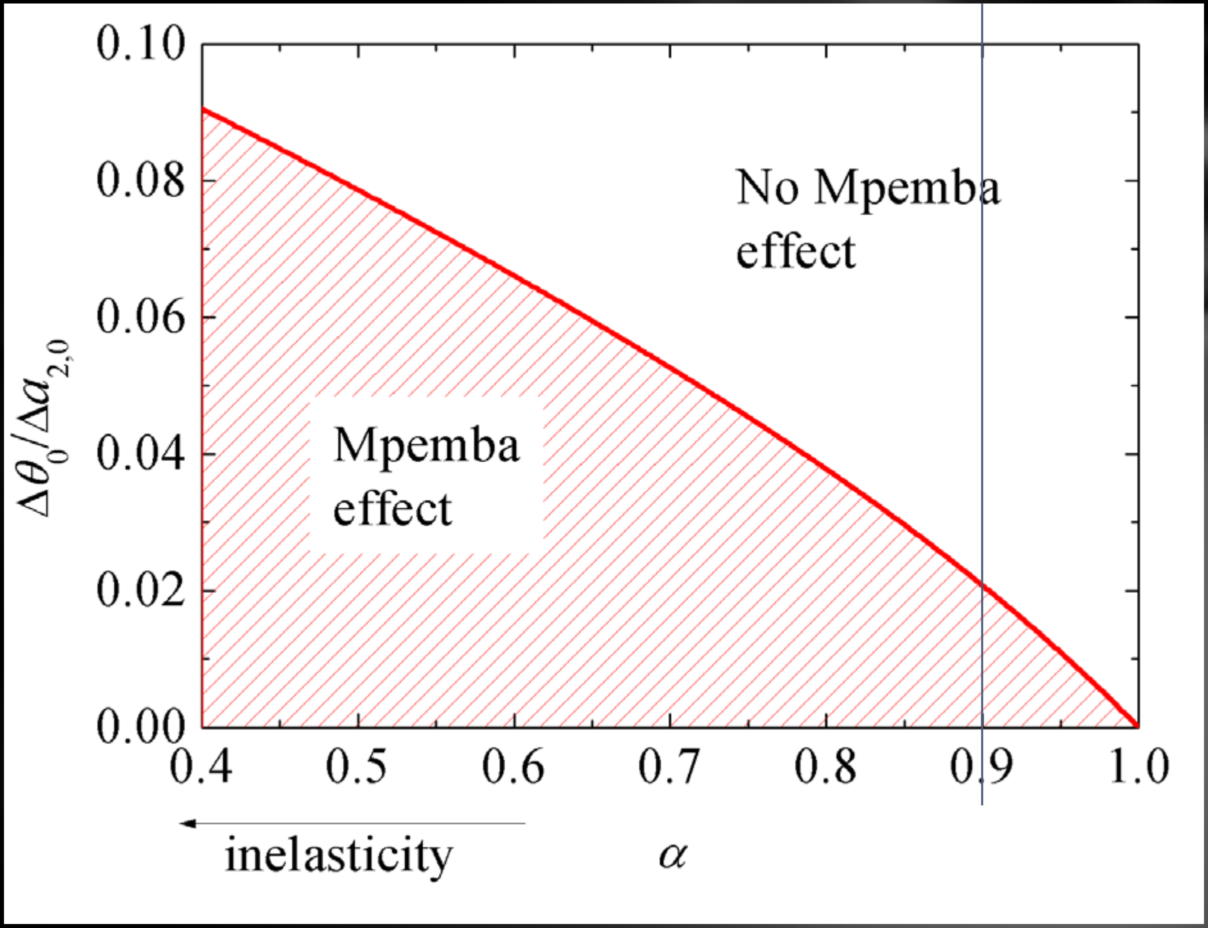
Initial condition A:

$$\theta_0 = \theta_A, a_{2,0} = a_{2A}$$

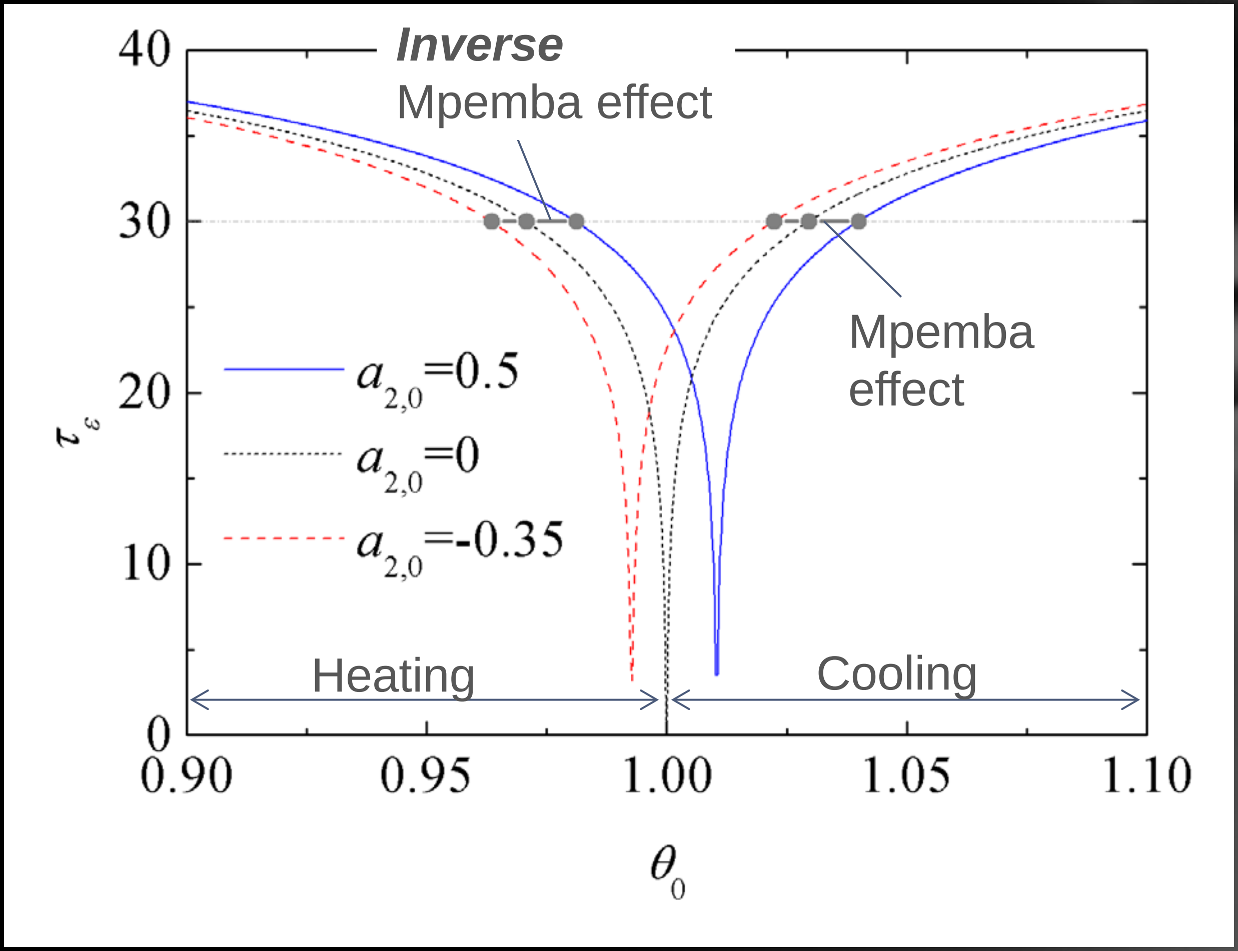
Initial condition B:

$$\theta_0 = \theta_B, a_{2,0} = a_{2B}$$

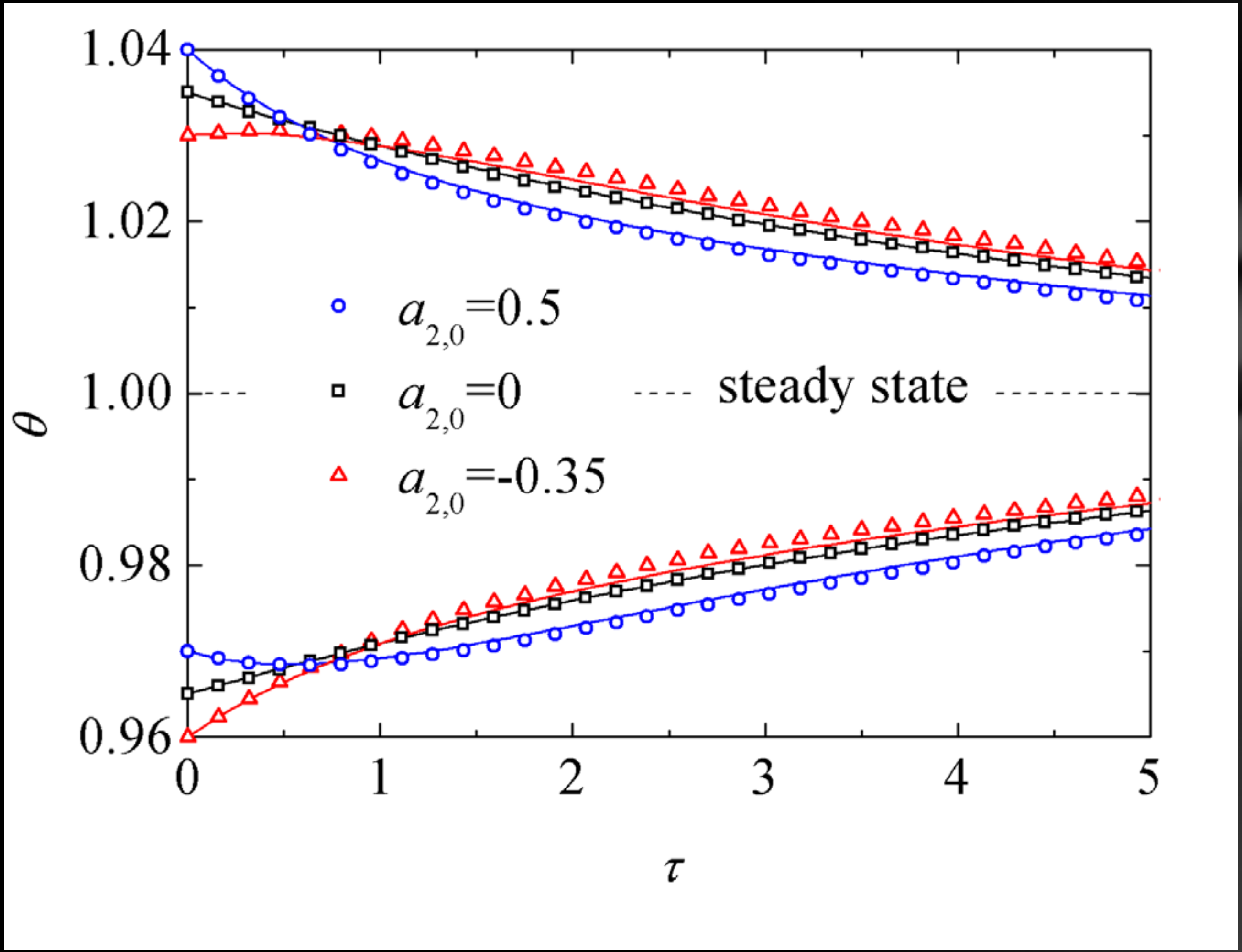
$$\Delta\theta_0 \equiv \theta_A - \theta_B$$
$$\Delta a_{2,0} \equiv a_{2A} - a_{2B}$$



Relaxation time to the steady state



Comparison with computer simulations (DSMC)



Mpemba effect

Inverse
Mpemba effect

Is the effect limited to $T_0 \sim T_s$?

"Newton-like" cooling equation: $\dot{T} = -\frac{2K}{3}\left(\cancel{\mu_2 T^{3/2}} - \cancel{\mu_{2,s} T_s^{3/2}}\right)$

1. $T_0 \ll T_s \Rightarrow \dot{T} \approx \frac{2K}{3}\mu_{2,s}T_s^{3/2} \Rightarrow$ NO (inverse) Mpemba effect

2. $T_0 \gg T_s \Rightarrow \dot{T} \approx -\frac{2K}{3}\mu_2 T^{3/2} \Rightarrow$ Possibility of Mpemba effect (homogeneous cooling state)

Phase diagram if $T_0 >> T_s$

Initial condition A:

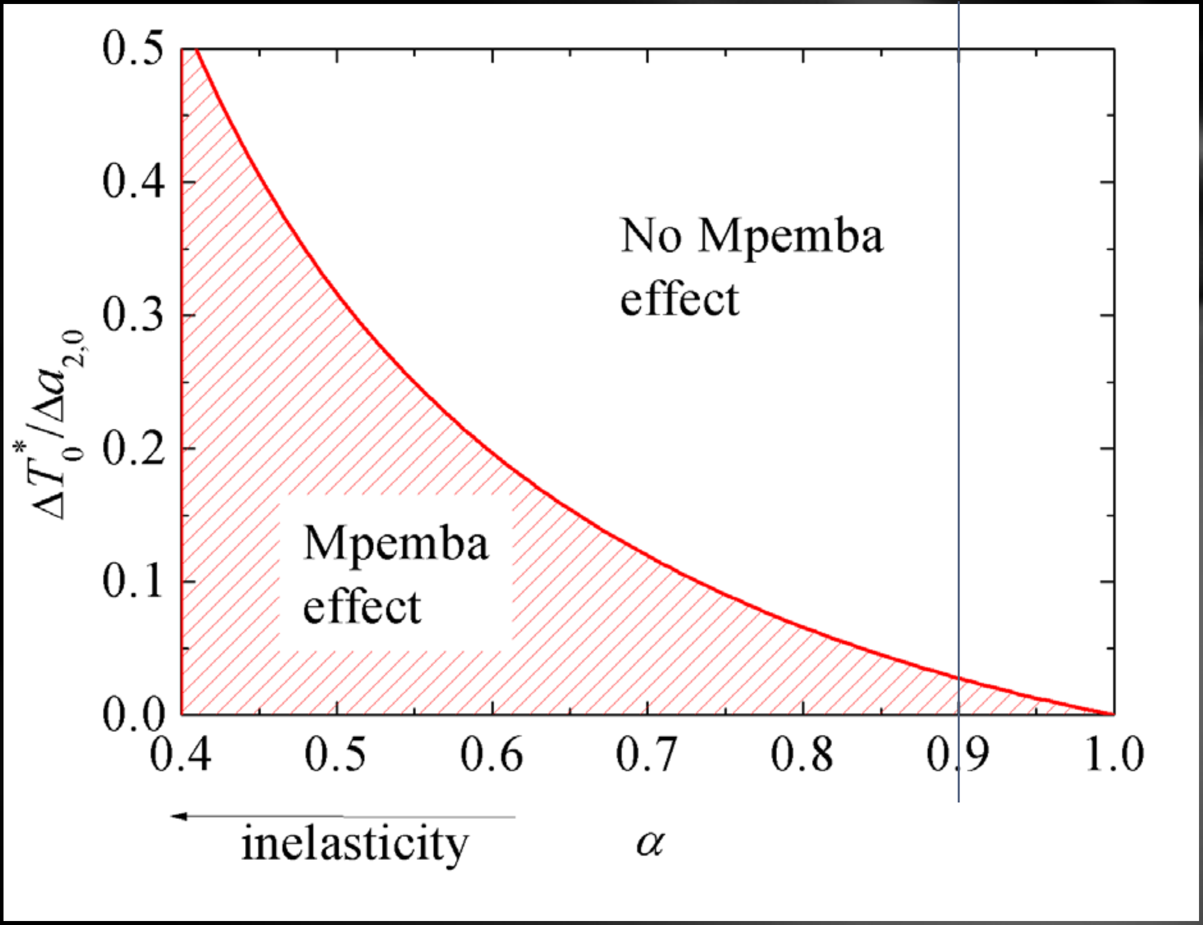
$$T_0 = T_A, a_{2,0} = a_{2A}$$

Initial condition B:

$$T_0 = T_B, a_{2,0} = a_{2B}$$

$$\Delta T_0^* \equiv \frac{T_A - T_B}{T_A}$$

$$\Delta a_{2,0} \equiv a_{2A} - a_{2B}$$



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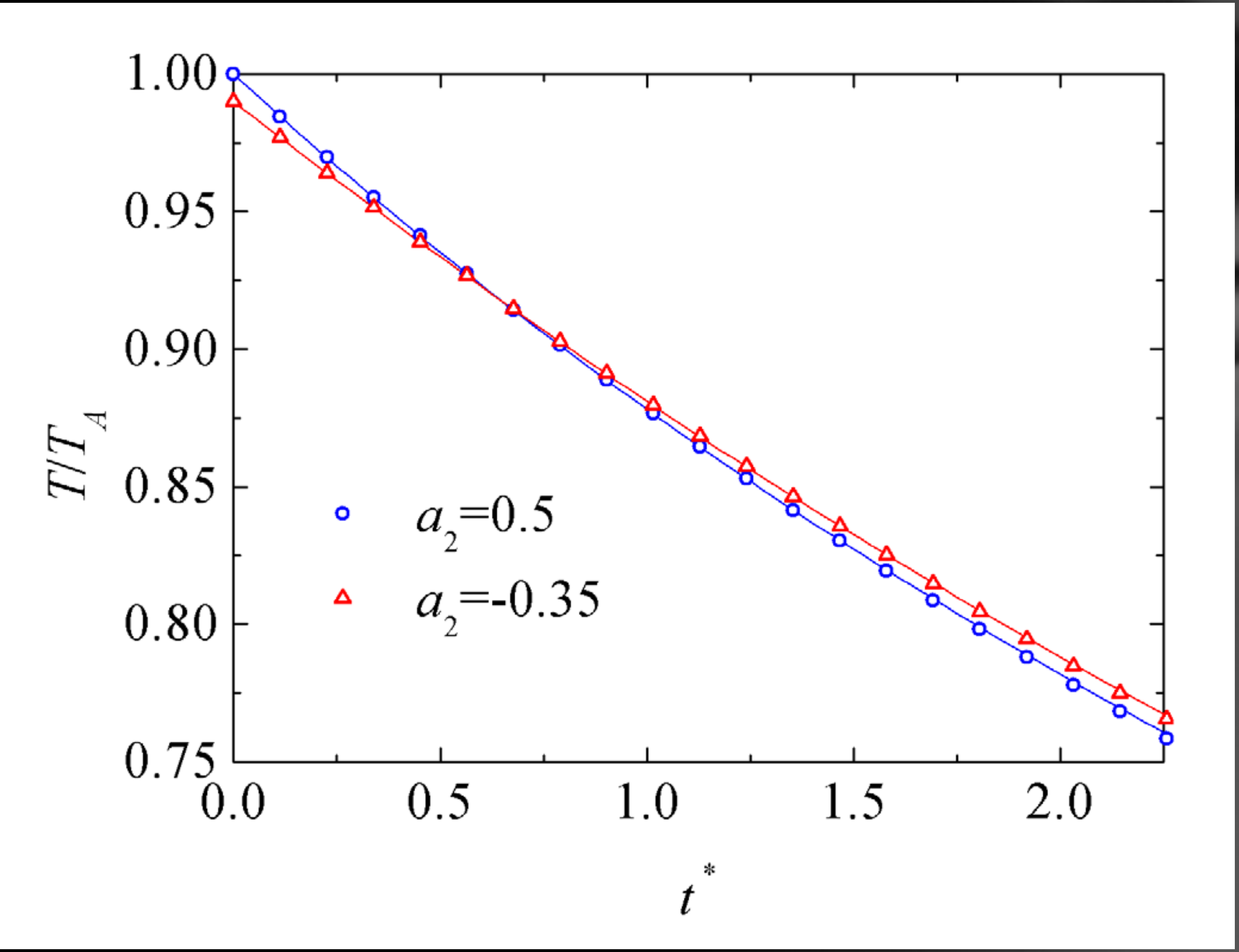

THE TAKE-HOME MESSAGE

- For a given system, the Mpemba effect can be expected if $dT/dt=F_T(T,\{X_j\})$.
- In a homogeneous granular gas, the simplest approach [$dT/dt=F_T(T,a_2)$, $da_2/dt=F_a(T,a_2)$] describes the effect very accurately.
- The effect can also exist in a molecular gas (elastic collisions) driven by a *nonlinear* drag.


THANK
YOU!


What people think about
during your conference talk
Hey! That's what I'm working on!
Hee hee! Animated arrows!
Typo on slide 14, line 3, centre-left. Noted.
(RANDOM HAPPY THOUGHTS]
I'm up next I'm up next I'm up next
I'm in the wrong session.
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