TRANSPORT PROPERTIES IN DISPARATE-MASS BINARY GASES

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We have studied the transport properties in disparate-mass binary gases. The Hilbert method is applied to the coupled BGK kinetic model equations. A set of two-fluid transport equations is derived to the Navier–Stokes hydrodynamic order. The results are compared with those obtained using Chapman–Enskog techniques.

1. Introduction

Transport phenomena in mixed binary gases are generally described by coupled Boltzmann equations [1]. The Chapman–Enskog (CE) expansion is the standard method of solution. The fundamental characteristic of this method to lowest order is that one obtains a velocity distribution function which is locally Maxwellian about a single temperature and velocity. Consequently, the mixture is described as a pure gas with a single temperature and flow velocity and a diffusion velocity is ascribed to each of its components. This is the essential difference with respect to the usual description of a pure mono-atomic gas.

However, for disparate-mass binary gases the conventional CE method is not adequate. One needs a description which includes the possibility of different temperatures for the different components of the binary mixture. In this situation it seems natural to take as the reference state (lowest order in the normal solution) local Maxwellians about species flow velocities and temperatures. In the last few years various studies have appeared with the aim of describing disparate-mass binary gases to Navier–Stokes order. Some of them are based on model kinetic Boltzmann equations [2–5]. Others make use of CE-type techniques with various collision integral reorderings [6–9] and the remainder use Grad's thirteen-moment [10–13] approach for a binary mixture of Maxwellian molecules.

The aim of this paper is the study of transport properties in disparate-mass binary gases. We start with the Bhatnagar–Gross–Krook (BGK) [14] kinetic equations for a gas mixture. Recently, the BGK equation has been solved by the Hilbert method [15,16] for the pure gas case. In this paper we apply Hilbert theory to the coupled BGK equations. Recently, Delale [17] has also used this method to solve the Boltzmann equations for gas mixtures. The present work differs from that of Delale.

We compute the pressure tensor and the heat flux vector within the Navier–Stokes approximation and the analysis is constrained to Maxwellian molecules. The first term in the Hilbert expansion corresponds to local Maxwellians for species flow velocities and temperatures which leads to a set of two-fluid transport equations. In this hydrodynamic order there is diffusion and so these equations are not the conventional Euler solutions for ideal flow. In the second hydrodynamic stage the transport equations are similar to those obtained using the coupled Boltzmann equations [7].

2. Kinetic model

We consider a binary gas mixture whose distribution functions $f_i(r, v; t)$ and $f_j(r, v; t)$ obey the coupled BGK equations [3]

$$(\partial/\partial t + v_\alpha \nabla_\alpha)f_i = -\zeta_{ii}(f_i - \phi_i) - \zeta_{ij}(f_i - \tilde{\phi}_i) \equiv J_{ii} + J_{ij} ,$$

$$(\partial/\partial t + v_\alpha \nabla_\alpha)f_j = -\zeta_{jj}(f_j - \phi_j) - \zeta_{ji}(f_j - \tilde{\phi}_j) \equiv J_{jj} + J_{ji} . \tag{1}$$

Throughout i and j are species indices while the Greek subscripts refer to vector components. In eq. (1), J_{ii} corresponds to the self-collision term of the gas i and J_{ij} to the cross-collision term, and similarly J_{jj} and J_{ji} for the gas j. Furthermore, we have introduced the parameters ζ_{ii}, the frequency of collisions of molecules i among themselves, ζ_{ij} the frequency of collisions between molecules i and j, and the functions ϕ_i, $\tilde{\phi}_i$ given by

$$\phi_i = n_i(m_i/2\pi k_B T_i)^{3/2} \exp[-(m_i/2k_B T_i)(v - u_i)^2] , \tag{2}$$

$$\tilde{\phi}_i = n_i(m_i/2\pi k_B \tilde{T}_i)^{3/2} \exp[-(m_i/2k_B \tilde{T}_i)(v - \tilde{u}_i)^2] . \tag{3}$$

The parameters n_i, u_i and T_i are the species number density, flow velocity and temperature, respectively,

$$n_i = \int dv f_i , \tag{4}$$

$$n_i u_i = \int dv v f_i , \tag{5}$$

$$\tfrac{3}{2} n_i k_B T_i = \int dv \tfrac{1}{2} m_i V_i^2 f_i , \tag{6}$$

where $V_i = v - u_i$ is the velocity of species i. The parameters with a tilde that appear in (3) will be computed later by applying conservation principles. From eqs. (4)–(6) we may define the mass density, flow velocity and temperature of the mixture by the relations:

$$\rho = \sum_r \int dv f_r = \sum_r m_r n_r = \sum_r \rho_r \quad (r \equiv i, j) , \tag{7}$$

$$\rho u = \sum_r \int dv m_r v f_r = \sum_r \rho_r u_r \quad (r \equiv i, j) , \tag{8}$$

$$\tfrac{3}{2} n k_B T = \sum_r \int dv \tfrac{1}{2} m_r (v - u)^2 f_r = \sum_r (\tfrac{3}{2} n_r k_B T_r + \tfrac{1}{2} \rho_r \omega_r^2) \quad (r \equiv i, j) , \tag{9}$$

where $\omega_r \equiv u_r - u$ is the diffusion velocity of species r. In order to obtain the moment equations in a two-fluid form [6], we take species mass, momentum and energy moments $(1, m_i V_i, \tfrac{1}{2} m_i V_i^2)$ of the coupled BGK equations (1):

$$(\partial/\partial t + u_{i\beta}\nabla_\beta)n_i + n_i \nabla_\beta u_{i\beta} = 0 , \tag{10}$$

$$\rho_i(\partial/\partial t + u_{i\beta}\nabla_\beta)u_{i\alpha} + \nabla_\beta P_{i\alpha\beta} = -\rho_i \zeta_{ij}(u_{i\alpha} - \tilde{u}_{i\alpha}) \equiv L_{i\alpha} , \tag{11}$$

$$\tfrac{3}{2} n_i k_B(\partial/\partial t + u_{i\beta}\nabla_\beta)T_i + P_{i\alpha\beta}\nabla_\beta u_{i\alpha} + \nabla_\beta q_{i\beta} = -\zeta_{ij}\tfrac{3}{2} n_i k_B(T_i - \tilde{T}_i) + \tfrac{1}{2}\rho_i \zeta_{ij}(u_i - \tilde{u}_i)^2 \equiv M_i , \tag{12}$$

where we have defined the momentum and heat fluxes with reference to the species flow velocities:

$$P_{i\alpha\beta} = \int dv m_i V_{i\alpha} V_{i\beta} f_i , \tag{13}$$

$$q_{i\alpha} = \int dv \tfrac{1}{2} m_i V_i^2 V_{i\alpha} f_i . \tag{14}$$

Obviously analogous expressions may be obtained for component j by replacing the subscript i by j. On the other hand, total momentum and energy conservation requires that

$$\sum_r L_r = 0, \quad \text{i.e.,} \quad m_i u_i + m_j u_j = m_i \tilde{u}_i + m_j \tilde{u}_j , \tag{15}$$

$$\sum_r M_r + u_r L_r = 0, \quad \text{i.e.} \quad 3k_B(T_i + T_j) + m_i u_i^2 + m_j u_j^2 = 3k_B(\tilde{T}_i + \tilde{T}_j) + m_i \tilde{u}_i^2 + m_j \tilde{u}_j^2 , \tag{16}$$

where we have considered $n_i \zeta_{ij} = n_j \zeta_{ji}$. In this way, one may easily obtain the familiar hydrodynamic conservation equations (see eqs. (6)–(8) of ref. [15]) although now the pressure tensor and the heat flux vector of the mixture will be given by

$$P_{\alpha\beta} = \sum_r \int d\mathbf{v}\, m_r V_{r\alpha} V_{r\beta} f_r = \sum_r (P_{r\alpha\beta} + \rho_r \omega_{r\alpha} \omega_{r\beta}) \quad (r \equiv i, j) , \tag{17}$$

$$q_\alpha = \sum_r \int d\mathbf{v}\, \tfrac{1}{2} m_r V_r^2 V_{r\alpha} f_r = \sum_r q_{r\alpha} + \sum_r \omega_{r\beta} P_{r\beta\alpha} + \sum_r \omega_{r\alpha} (\tfrac{1}{2}\rho_r \omega_r^2 + \tfrac{3}{2} n_r k_B T_r) . \tag{18}$$

In order to complete the model it is still necessary to compute the collision frequencies (subject to condition (15)) and to evaluate the parameters $\tilde{u}_i$, $\tilde{u}_j$, $\tilde{T}_i$ and $\tilde{T}_j$ (subject to conditions (16) and (17)). For that, we shall restrict ourselves to the particular case of Maxwell molecules (repelling each other with forces proportional to r^{-5}) for which [18] $\zeta_{ii} = \mu_i n_i$ and $\zeta_{ij} = \nu n_j$. In addition, if the potential parameters are the same for the three possible interactions, then $\mu_i \propto \sqrt{m_i}$ and $\nu \propto \sqrt{m_i + m_j}$. On the other hand, we expect that the most relevant features of the coupled Boltzmann equations will be reasonably well mimicked by the proposed kinetic model. Thus, we demand that the ratio of the momentum difference relaxation time to the temperature difference relaxation time obtained from eqs. (1) be the same as that of the Boltzmann equations for a binary Maxwell mixture [3]. Therefore,

$$\tilde{u}_i = \tilde{u}_j = \tilde{u} \equiv (m_i u_i + m_j u_j)/(m_i + m_j) , \tag{19}$$

$$\tilde{T}_i = T_i + 2[m_i m_j/(m_i + m_j)^2][(T_j - T_i) + (m_j/6k_B)(u_j - u_i)^2] , \quad i \leftrightarrow j . \tag{20}$$

3. Transport properties. Hilbert theory

As noted in section 1, we are interested in transport phenomena in disparate-mass binary gases of Maxwellian molecules; in particular, for the so-called Lorentzian mixtures for which $m_i \ll m_j$ and $n_i \ll n_j$. In this case, as a possible way of obtaining normal solutions to the BGK equations (1), we consider a reordering procedure due to Grad (see section 8 of ref. [6]). Following this procedure, the coupled BGK equations can be written as:

$$(\partial/\partial t + v_\alpha \nabla_\alpha) f_i = -\epsilon^{-1} \zeta_{ii} (f_i - \phi_i) - \zeta_{ij} (f_i - \tilde{\phi}_i) ,$$

$$(\partial/\partial t + v_\alpha \nabla_\alpha) f_j = -\zeta_{ji} (f_j - \tilde{\phi}_j) - \epsilon^{-1} \zeta_{jj} (f_j - \phi_j) , \tag{21}$$

where $\epsilon \equiv (m_i/m_j)^{1/2}$ is the uniformity parameter. We look for normal solutions to (21) by means of Hilbert theory [15] expanding the distribution functions in powers of ϵ

$$f_i = \sum_{k=0}^{\infty} \epsilon^k f_i^{(k)} , \tag{22}$$

and similarly for the hydrodynamic variables. When we introduce these expansions in (21), and separate terms order-by-order we obtain the algebraic equations

$$f_i^{(0)} = \phi_i^{(0)} ,$$

$$f_i^{(1)} = \phi_i^{(1)} - \frac{1}{\zeta_{ii}^{(0)}} v_\alpha \nabla_\alpha f_i^{(0)} - \frac{\zeta_{ij}^{(0)}}{\zeta_{ii}^{(0)}} (f_i^{(0)} - \tilde{\phi}_i^{(0)}) ,$$

$$\vdots$$

$$f_i^{(k)} = \phi_i^{(k)} - \frac{1}{\zeta_{ii}^{(0)}} v_\alpha \nabla_\alpha f_i^{(k-1)} - \frac{1}{\zeta_{ii}^{(0)}} \sum_{l=1}^{k-1} \zeta_{ii}^{(l)} (f_i^{(k-l)} - \phi_i^{(k-l)}) - \frac{1}{\zeta_{ii}^{(0)}} \sum_{l=1}^{k} \zeta_{ij}^{(l-1)} (f_i^{(k-l)} - \tilde{\phi}_i^{(k-l)}) \tag{23}$$

and identically for f_j. Note that the first terms on the right-hand side of eqs. (23) are identical to those found in the pure gas case [15]. These terms represent the self-collisions of a gas species. The last term involves the velocity and temperature differences of the two gases and represents cross-collisions between the two gases. From now on, in order to avoid having to specify the initial conditions we restrict the discussion to the steady state.

At zeroth order, we obtain velocity distribution functions $f_i^{(0)}$ which are local Maxwellians defined in terms of the flow velocity $u_i^{(0)}$ and temperature $T_i^{(0)}$,

$$f_i^{(0)} = n_i^{(0)} (m_i/2\pi k_B T_i^{(0)})^{3/2} \exp[-(m_i/2k_B T_i^{(0)}) \, V_i^{(0)2}] \,, \tag{24}$$

where $n_i^{(0)}$, $u_i^{(0)}$ and $T_i^{(0)}$ are defined in the usual manner. From (24), we obtain the two-fluid zeroth-order transport equations,

$$\nabla_\alpha(\rho_i^{(0)} u_{i\alpha}^{(0)}) = 0 \,, \tag{25}$$

$$\rho_i^{(0)} u_{i\beta}^{(0)} \nabla_\beta u_{i\alpha}^{(0)} + \nabla_\alpha p_i^{(0)} = n_i^{(0)} \zeta_{ij}^{(0)} \mu W_\alpha^{(0)} \equiv L_{i\alpha}^{(0)} \,, \tag{26}$$

$$\tfrac{3}{2} n_i^{(0)} k_B u_{i\beta}^{(0)} \nabla_\beta T_i^{(0)} + p_i^{(0)} \nabla_k u_{ik}^{(0)} = n_i^{(0)} \zeta_{ij}^{(0)} (\mu^2/m_i) \, (3k_B \theta^{(0)}/m_j + W^{(0)2}) \equiv M_i^{(0)} \,. \tag{27}$$

Here $\mu \equiv m_i m_j/(m_i + m_j)$ is the reduced mass, $W^{(0)} \equiv u_j^{(0)} - u_i^{(0)}$ is the diffusion velocity of heavy species with respect to the light species in this approximation, $\theta^{(0)} \equiv T_j^{(0)} - T_i^{(0)}$ and $p_i^{(0)} = n_i^{(0)} k_B T_i^{(0)}$. According to equation (26), in our description diffusion appears at the lowest order in the perturbation expansion. This is not the case in the conventional Hilbert [17] or CE [1] expansions because the species flow velocity $u_i^{(0)}$ coincides with the mixture flow velocity $u^{(0)}$ in the Euler regime. This is an essential difference between both descriptions. On the other hand, the mixture momentum and heat fluxes (eqs. (17), (18)) are given by

$$P_{\alpha\beta}^{(0)} = p^{(0)} \delta_{\alpha\beta} + \sum_r \rho_r^{(0)} \langle \omega_{r\alpha}^{(0)} \omega_{r\beta}^{(0)} \rangle \quad (r \equiv i, j) \,, \tag{28}$$

$$q_\alpha^{(0)} = \sum_r \omega_{r\alpha}^{(0)} (\tfrac{5}{2} n_r^{(0)} k_B T_r^{(0)} + \tfrac{1}{2} \rho_r^{(0)} \omega_r^{(0)2}) \quad (r \equiv i, j) \,. \tag{29}$$

Here $p^{(0)} = n^{(0)} k_B T^{(0)}$, and $\langle A_\alpha B_\beta \rangle = \tfrac{1}{2}(A_\alpha B_\beta + A_\beta B_\alpha) - \tfrac{1}{3}\delta_{\alpha\beta} A_k B_k$ is the symmetric traceless part of the tensor AB.

At the Navier–Stokes order, after some manipulation, the function $f_i^{(1)}$ may be written as,

$$f_i^{(1)} = f_i^{(0)} \left[\frac{n_i^{(1)}}{n_i^{(0)}} + \left(\frac{m_i V_i^{(0)2}}{2k_B T_i^{(0)}} - \frac{3}{2} \right) \frac{T_i^{(1)}}{T_i^{(0)}} + \frac{m_i}{k_B T_i^{(0)}} V_{i\alpha}^{(0)} u_{i\alpha}^{(1)} - \frac{V_{i\alpha}^{(0)}}{\zeta_{ii}^{(0)}} \left(\frac{m_i V_i^{(0)2}}{2k_B T_i^{(0)}} - \frac{5}{2} \right) \frac{\nabla_\alpha T_i^{(0)}}{T_i^{(0)}} \right.$$

$$\left. - \frac{m_i}{k_B T_i^{(0)} \zeta_{ii}^{(0)}} \langle V_{i\alpha}^{(0)} V_{i\beta}^{(0)} \rangle \nabla_\beta u_{i\alpha}^{(0)} - \frac{V_{i\alpha}^{(0)} L_{i\alpha}^{(0)}}{p_i^{(0)} \zeta_{ii}^{(0)}} - \frac{2}{3} \frac{M_i^{(0)}}{p_i^{(0)} \zeta_{ii}^{(0)}} \left(\frac{m_i V_i^{(0)2}}{2k_B T_i^{(0)}} - \frac{3}{2} \right) \right] - \frac{\zeta_{ij}^{(0)}}{\zeta_{ii}^{(0)}} (f_i^{(0)} - \tilde{\phi}_i^{(0)}) \,, \tag{30}$$

where $n_i^{(1)}$, $u_i^{(1)}$ and $T_i^{(1)}$ are defined from $f_i^{(1)}$. In order to obtain the corresponding set of two-fluid equations at the Navier–Stokes level, we need to calculate the species pressure tensor and heat flux vector:

$$P_{i\alpha\beta}^{(1)} = \int \mathrm{d}v \, m_i V_{i\alpha}^{(0)} V_{i\beta}^{(0)} f_i^{(1)} = p_i^{(1)} \delta_{\alpha\beta} - 2\eta_i \langle \nabla_\alpha u_{i\beta}^{(0)} \rangle + B_i \langle W_\alpha^{(0)} W_\beta^{(0)} \rangle \,, \tag{31}$$

$$q_{i\alpha}^{(1)} = \int \mathrm{d}v \, \tfrac{1}{2} m_i V_i^{(0)2} V_{i\alpha}^{(0)} f_i^{(1)} - \tfrac{5}{2} p_i^{(0)} u_{i\alpha}^{(1)} = -\lambda_i \nabla_\alpha T_i^{(0)} + A_i W_\alpha^{(0)} \,, \tag{32}$$

where

$$p_i^{(1)} = n_i^{(1)} k_B T_i^{(0)} + n_i^{(0)} k_B T_i^{(1)} \quad \text{and} \quad \eta_i = p_i^{(0)}/\zeta_{ii}^{(0)} \,, \quad \lambda_i = \tfrac{5}{2} p_i^{(0)} k_B/m_i \zeta_{ii}^{(0)}$$

are the pure gas shear viscosity and thermal conductivity coefficients evaluated at the species variables. Additionally, we have introduced the parameters

$$B_i \equiv n_i^{(0)} \mu^2 \zeta_{ij}^{(0)} / m_i \zeta_{ii}^{(0)} \quad \text{and} \quad A_i \equiv (n_i^{(0)} \mu^3 \zeta_{ij}^{(0)} / m_i^2 \zeta_{ii}^{(0)})(5 k_B \theta^{(0)}/m_j + \tfrac{4}{3} W^{(0)2}).$$

All the cross-collisional effects appear in the last terms of the right-hand side of eqs. (31) and (32). These results are similar to those obtained from the Boltzmann equations in the two-temperature description [7].

If we introduce (31) and (32) in the moment equations (10)–(12), we obtain the two-fluid Navier–Stokes equations,

$$\nabla_\alpha [n_i^{(0)} u_{i\alpha}^{(1)} + n_i^{(1)} u_{i\alpha}^{(0)}] = 0, \tag{33}$$

$$\nabla_\alpha [\rho_i^{(0)} u_{i\alpha}^{(0)} u_{i\beta}^{(1)} + \rho_i^{(0)} u_{i\alpha}^{(1)} u_{i\beta}^{(0)} + \rho_i^{(1)} u_{i\alpha}^{(0)} u_{i\beta}^{(0)} + p_i^{(1)} \delta_{\alpha\beta} - 2\eta_i \langle \nabla_\alpha u_{i\beta}^{(0)} \rangle + B_i \langle W_\alpha^{(0)} W_\beta^{(0)} \rangle]$$

$$= (n_i^{(1)} \zeta_{ij}^{(0)} + n_i^{(0)} \zeta_{ij}^{(0)}) \mu W_\alpha^{(0)} + n_i^{(0)} \zeta_{ij}^{(0)} \mu W_\alpha^{(1)}, \tag{34}$$

$$\tfrac{3}{2} n_i^{(0)} k_B [u_{i\beta}^{(0)} \nabla_\beta T_i^{(1)} + u_{i\beta}^{(1)} \nabla_\beta T_i^{(0)}] + p_i^{(0)} \nabla_k u_{ik}^{(1)} + n_i^{(0)} k_B T_i^{(1)} \nabla_k u_{ik}^{(0)} - \eta_i (\nabla_\alpha u_{i\beta}^{(0)} \nabla_\beta u_{i\alpha}^{(0)}$$

$$+ \nabla_\beta u_{i\alpha}^{(0)} \nabla_\beta u_{i\alpha}^{(0)}) + \tfrac{2}{3} \eta_i (\nabla_k u_{ik}^{(0)})^2 + B_i \langle W_\alpha^{(0)} W_\beta^{(0)} \rangle \nabla_\beta u_{i\alpha}^{(0)} - \nabla_\alpha (\lambda_i \nabla_\alpha T_i^{(0)}) + \nabla_\alpha (A_i W_\alpha^{(0)})$$

$$= \frac{3 n_i^{(0)} k_B}{m_i m_j} \mu^2 (\zeta_{ij}^{(0)} \theta^{(1)} + \zeta_{ij}^{(1)} \theta^{(0)}) + \frac{n_i^{(0)} \mu^2}{m_i} (2\zeta_{ij}^{(0)} W_\alpha^{(0)} W_\alpha^{(1)} + \zeta_{ij}^{(1)} W^{(0)2}), \tag{35}$$

where obviously $W^{(1)} \equiv u_j^{(1)} - u_i^{(1)}$ and $\theta^{(1)} \equiv T_j^{(1)} - T_i^{(1)}$. These equations constitute a set of linear partial differential equations for the species variables $n_i^{(1)}$, $u_i^{(1)}$ and $T_i^{(1)}$, which can be solved from the first approximation solution. From (31) and (32), one can easily derive the mixture transport properties. Then, after some algebra, the pressure tensor and the heat flux vector of the mixture will be given by

$$P_{\alpha\beta}^{(1)} = p^{(1)} \delta_{\alpha\beta} - \sum_r [2\eta_r \langle \nabla_\alpha u_{r\beta}^{(0)} \rangle - B_r \langle W_\alpha^{(0)} W_\beta^{(0)} \rangle - 2\rho_r^{(0)} \langle \omega_{r\alpha}^{(0)} \omega_{r\beta}^{(1)} \rangle - \rho_r^{(1)} \langle \omega_{r\alpha}^{(0)} \omega_{r\beta}^{(0)} \rangle] \quad (r \equiv i, j),$$

$$\tag{36}$$

$$q_\alpha^{(1)} = - \sum_r [\lambda_r \nabla_\alpha T_r^{(0)} - \tfrac{5}{2} (p_r^{(0)} \omega_{r\alpha}^{(1)} + p_r^{(1)} \omega_{r\alpha}^{(0)}) + 2\eta_r \omega_{r\alpha} \langle \nabla_\alpha u_{r\beta}^{(0)} \rangle - B_r \omega_{r\alpha}^{(0)} \langle W_\alpha^{(0)} W_\beta^{(0)} \rangle$$

$$- \tfrac{1}{2} \omega_r^{(0)2} (\rho_r^{(1)} \omega_{r\alpha}^{(0)} + \rho_r^{(0)} \omega_{r\alpha}^{(1)}) - \rho_r^{(0)} \omega_{r\alpha}^{(0)} \omega_{r\beta}^{(0)} \omega_{r\beta}^{(1)}] + (A_i - A_j) W_\alpha^{(0)} \quad (r \equiv i, j), \tag{37}$$

where $p^{(1)} = n^{(1)} k_B T^{(0)} + n^{(0)} k_B T^{(1)}$. According to (34) and (37), one may observe the cross transport that appears in the gas mixture: thermal diffusion and the diffusion thermoeffect. In this way, one may define the corresponding transport coefficients of the binary mixture.

In summary, we have analyzed the transport properties in disparate-mass binary gases by means of the coupled BGK equations. The solution of the BGK model is worked out by the Hilbert method. The first term in the series expansion of the distribution function are shown to be local Maxwellians about the independent flow velocities and temperatures. The zeroth-order set of two-fluid transport equations does not correspond to Euler equations of ideal flow. There is diffusion in this approximation. The pressure tensor and heat flux vector are computed to Navier–Stokes order. The results are identical to those obtained from the Boltzmann equations for disparate-mass binary gases.

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