Rheological Properties of a Granular Impurity in the Couette Flow

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Abstract. We discuss in this work the validity of the theoretical solution of the nonlinear Couette flow for a granular impurity obtained in a recent work [preprint arXiv:0802.0526], in the range of large inelasticity and shear rate. We show there is a good agreement between the theoretical solution and Monte Carlo simulation data, even under these extreme conditions. We also discuss an extended theoretical solution that would work for large inelasticities in ranges of shear rate a not covered by our previous work (i.e., below the threshold value a_{th} for which uniform shear flow may be obtained) and compare also with simulation data. Preliminary results in the simulations give useful insight in order to obtain an exact and general solution of the nonlinear Couette flow (both for $a \geq a_{\text{th}}$ and $a < a_{\text{th}}$).

Keywords: Granular gases; Boltzmann equation; Couette flow; Rheological properties
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Granular materials are generically characterized by the inelasticity in the collisions between the particles they are composed of. Transport theories of fluidized granular materials have been the subject of a considerable amount of research work in the last few years [1, 2]. We may find two important reasons motivating this effort: 1) the transport of granular fluids has numerous and obvious industrial applications, and 2) the theories and experiments on granular media constitute, in several important cases, generalizations of existing theories on elastic fluids [1, 2]. Additionally, computational tools used for elastic fluids such as molecular dynamics (MD) [3] and Monte Carlo (DSMC) simulations [4] may also be used in the realm of inelastic collisions [1, 2], and for this reason the study of granular media is an open and challenging field also for computational physicists interested in non-equilibrium systems [1]. We may illustrate the idea of generalization of previous theories with the help of a paradigmatic example: in the smooth hard sphere model, inelastic collisions are characterized by the coefficient of normal restitution α, whose value in the particular case of elastic collisions is unity [2]. For instance, the recent calculations of transport coefficients of granular gases for a monocomponent gas [5], binary dilute mixtures [6], and the more realistic case of multicomponent dense mixtures [7] are based on the extension of the Chapman–Enskog method to inelastic gases, and this is done by taking into account inelasticity (i.e., the possibility of having $\alpha \neq 1$) in the collisional integrals of the corresponding kinetic equation [5]. The multiplicity of behaviors of granular systems [1] emerges as a very important consequence of the fact that the study of granular gases is a generalization of that for elastic gases: for example, while for elastic gases the steady Couette flow is characterized by a parameter $\gamma > 0$, for granular gases the three possibilities $\gamma < 0, \gamma = 0, \gamma > 0$ can occur (we will later describe the meaning of this parameter), which results in having up to 6 types of steady Couette flows in granular gases whereas in elastic gases only one is possible [8]. Of course this complexity in granular media arises also in their rheology, whose study is particularly interesting because nonlinearity is inherent to granular rapid flows [9], except in the quasielastic limit [8]. We will analyze in this work the rheology of a granular impurity under Couette flow, focusing on conditions of large inelasticity and hydrodynamic gradients. We will use for this the exact solution of the nonlinear Couette flow of a granular impurity, for the case $\gamma \geq 0$, recently developed by the authors [10] and we will show, with the help of DSMC data, that our theoretical solution works well even under these extreme conditions. Nevertheless, our theoretical solution only covers the cases $a \geq a_{\text{th}}$, where a_{th} is the value for which the well known uniform shear flow may be obtained [9], while the real situation is not restricted to this possibility, as simulation data we present now clearly show. For this reason, we will also explore in this work the possibility of extending the solution to the case $\gamma < 0$, which occurs for $a < a_{\text{th}}$.

This work is motivated by a recent previous work [10], where we obtained the exact solution of the nonlinear Couette flow of a granular impurity for a BGK-type kinetic model [11], suitably adapted to the granular binary mixture with the addition of a drag volume force [12]. We assumed in our previous work [10] that the impurity (species 1) has the same flow velocity as the excess component (species 2), i.e., $\mathbf{u}_1 = \mathbf{u}_2$, and that the concentration ratio n_1/n_2 and the temperature ratio T_1/T_2 are spatially uniform. In addition, the hydrodynamic profiles of the excess component [9]

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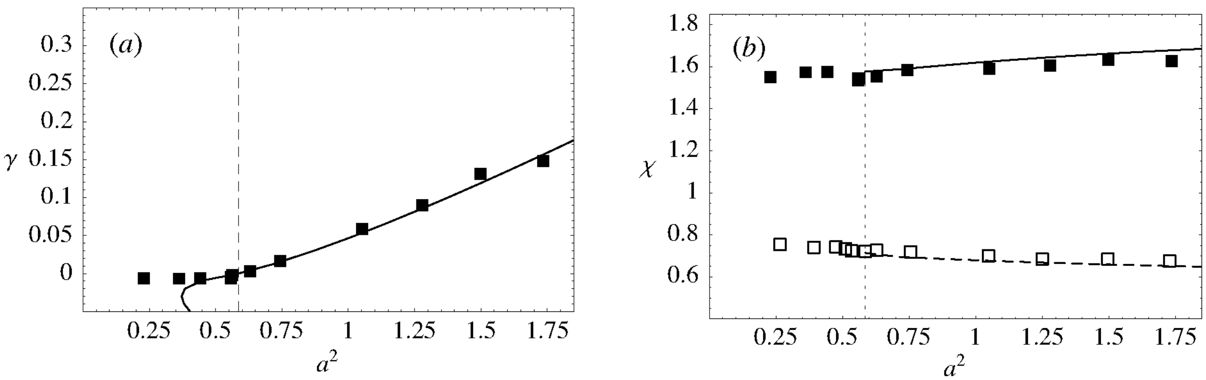


FIGURE 1. (*a*) The curvature parameter γ for the excess component as a function of the reduced shear rate squared a^2. The vertical dashed line indicates the threshold value a^2_{th} separating the regions with $\gamma < 0$ (to the left) and $\gamma > 0$ (to the right). The continuous line stands for the theoretical solution, and the points for DSMC data. The extended theoretical solution for $\gamma < 0$ works reasonably well in a finite interval of a to the left of a_{th}. (*b*) The temperature ratio χ vs. a^2 for a granular impurity with $\alpha = 0.5$, a unit size ratio, and mass ratios $m_1/m_2 = 2$ (continuous line, solid symbols) and $m_1/m_2 = 0.5$ (dashed line, open symbols). Lines and symbols stand for theoretical and DSMC data, respectively. The behavior of χ clearly indicates energy non-equipartition even without shear ($a \to 0$).

are an extension of those characterizing the nonlinear Couette flow for the elastic gas [11]. Thus, the hydrodynamic profiles have the following form:

$$\frac{1}{\nu_2(y)}\frac{\partial}{\partial y}u_{2,x} = a = \text{const}, \quad \frac{1}{2m_2}\left[\frac{1}{\nu_2(y)}\frac{\partial}{\partial y}\right]^2 T_2 = -\gamma = \text{const}, \quad n_2 T_2 = \text{const}, \tag{1}$$

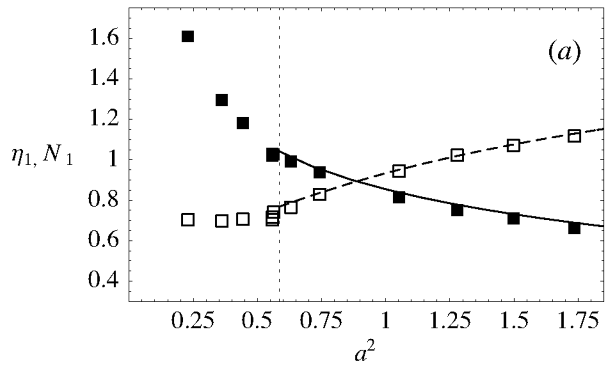
where the x axis coincides with the direction of the flow, the walls of the system are parallel to it and perpendicular to the y axis, γ is the curvature parameter for the excess component temperature, and $\nu_2(y) \propto n_2(y)\sqrt{T_2(y)}$ is an effective collision frequency [10]. The solution provides γ, the temperature ratio $\chi \equiv T_1/T_2$, and the rheological properties of both species as functions of the reduced shear rate a and the mechanical parameters (size ratio, mass ratio, and coefficients of restitution). In the solution use is made of the characteristic function $F_{0,m}(\gamma, \zeta_2/\nu_2)$ (where ζ_2 is the cooling rate of species 2) given by [10]

$$F_{0,m}(y,z) = \int_0^\infty dw\, e^{-(1+z)w} w^m \left[\sqrt{\pi}\theta(w,y,z)e^{\theta^2(w,y,z)}\text{erfc}\left(\theta(w,y,z)\right) - 1\right], \quad \theta(w,y,z) \equiv \frac{1}{2\sqrt{2y}}\frac{z}{1-e^{-\frac{1}{2}zw}}. \tag{2}$$

Note that $F_{0,m}(\gamma, \zeta_2/\nu_2)$ is well-defined for $\gamma \geq 0$ only. As said before, the case $\gamma = 0$ corresponds to a threshold value a_{th} of the shear rate at which the symmetric Couette flow reduces to the uniform shear flow [10, 9]. On the other hand, one can construct an *analytical continuation* of the function $F_{0,m}(y,z)$ for negative y values as

$$F_{0,m}(y,z) = \int_0^\infty dw\, e^{-(1+z)w} w^m \left[\sqrt{\pi}\widetilde{\theta}(w,y,z)e^{-\widetilde{\theta}^2(w,y,z)}\text{erfi}\left(\widetilde{\theta}(w,y,z)\right) - 1\right], \quad \widetilde{\theta}(w,y,z) \equiv \frac{1}{2\sqrt{-2y}}\frac{z}{1-e^{-\frac{1}{2}zw}}. \tag{3}$$

We proved the hypotheses $\mathbf{u}_1 = \mathbf{u}_2$, $n_1/n_2 = \text{const}$, $T_1/T_2 = \text{const}$, and Eq. (1) to be accurate by numerically solving (DSMC) our model kinetic equations, at least for values of the coefficient of normal restitution as low as $\alpha = 0.8$ and values of the reduced shear rate as high as $a \sim 1$ [10]. We present in this work results for considerably larger inelasticities ($\alpha = 0.5$) and shear rates (up to $a = 1.75$). Additionally, we present preliminary results, for the excess component, of an approximate solution based on Eq. (3) of the nonlinear Couette flow in the $\gamma < 0$ region that we are currently working out. As we see in Figs. 1 and 2, the results are quite satisfactory: in Fig. 1(*a*) we can notice that the extended solution works well for $\gamma(a)$ in a finite range of values of $a < a_{\text{th}}$, although the solution evidently fails below a critical value of a, and that the regular solution for $a \geq a_{\text{th}}$ ($\gamma \geq 0$) shows an excellent agreement with DSMC data. It is also to be noticed the good agreement for the temperature ratio $\chi = T_1/T_2$, that, as it is known and we can see in the figure, in granular mixtures is different from 1 even in the absence of shear ($a = 0$). In Fig. 2 we present the results corresponding to the stress tensor transport coefficients of the impurity. All of them show a very



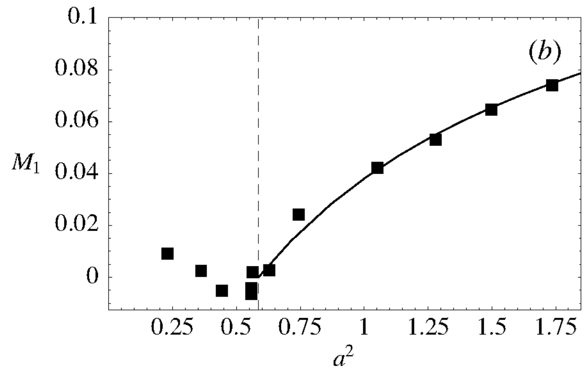


FIGURE 2. (*a*) Viscosity η_1 (continuous line, solid symbols) and normal stress difference N_1 (dashed line, open symbols) coefficients vs. a^2 for a granular impurity with $\alpha = 0.5$, a unit size ratio, and a mass ratio $m_1/m_2 = 2$. (*b*) Normal stress difference coefficient M_1 vs. a^2 for the same granular impurity. Lines stand for theory and points for simulation data.

good agreement with the numerical solution: the non-Newtonian shear viscosity η_1, and the normal stress differences $N_1 = (P_{1,xx} - P_{1,yy})/p_1$, $M_1 = (P_{1,zz} - P_{1,yy})/p_1$, with $p_1 = n_1 T_1$, whose magnitude shows the importance of rheological effects in this system. We can also notice in Figs. 1 and 2 that, independently of the limitation of our theory to the case $a \geq a_{\text{th}}$, the numerical solution of the kinetic equation (simulation data) extends beyond that limit, and the functions represented do not show any discontinuity in their behavior (except perhaps M_1) at $a = a_{\text{th}}$.

Summarizing, we have shown in this work that the theoretical solution of the nonlinear Couette flow obtained in a previous work [10] can be safely used for large inelasticities ($\alpha = 0.5$) and hydrodynamic gradients ($a \sim 1.75$). We also checked that the hypotheses underlying the theoretical solution, as well as its predictions for the rheological properties, are fulfilled for this large inelasticity. We have also discussed an extension of this theoretical solution in the region of shear rates below the threshold value $a < a_{\text{th}}$, i.e., negative value of the curvature parameter γ. These results are also useful for a future work, where we will assess to which degree our BGK-type kinetic model is able to describe the results from the 'true' Boltzmann kinetic equations.

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