# Some Computations in Cosmology

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In this notebook we present some useful routines for certain computations in Cosmology.

Let us define the Hubble's constant $H_0$.

[1]:
```
Hzero =  1/(9.78*10^9) #years.
```

We are interested in computing the age of the Universe for given values of $\Omega_\Lambda$, $\Omega_M$, $\Omega_R$ and $h$, where we will take into account

$$1 = \Omega_\Lambda + \Omega_R + \Omega_M + \Omega_k,$$

being $\Omega_k$ the curvature density parameter, that for a flat spacetime is equal to 0. Therefore, the age of the Universe is given by

$$t_U = \frac{1}{hH_0}\int_0^1 dx\, \frac{1}{x\sqrt{\Omega_\Lambda + \Omega_M x^{-3} + \Omega_R x^{-4} + (1-\Omega_\Lambda-\Omega_R-\Omega_M)x^{-2}}}.$$

[2]:
```
def AgeUniverse(OmegaLambda,OmegaM,OmegaR,h):
    Omegak = 1-OmegaLambda-OmegaM-OmegaR
    return numerical_integral(1/(x*sqrt(OmegaLambda+OmegaM/x^3+OmegaR/x^4+Omegak/
↪x^2)),0,1)[0]/(h*Hzero)
```

If we want to compute the age of the Universe for a given redshift $z$,

$$t_U(z) = \frac{1}{hH_0}\int_0^{1/(1+z)} dx\, \frac{1}{x\sqrt{\Omega_\Lambda + \Omega_M x^{-3} + \Omega_R x^{-4} + (1-\Omega_\Lambda-\Omega_R-\Omega_M)x^{-2}}}.$$

[3]:
```
def AgeUniverseRedshift(OmegaLambda,OmegaM,OmegaR,h,z):
    Omegak = 1-OmegaLambda-OmegaM-OmegaR
    return numerical_integral(1/(x*sqrt(OmegaLambda+OmegaM/x^3+OmegaR/x^4+Omegak/
↪x^2)),0,1/(1+z))[0]/(h*Hzero)
```

[4]:
```
AgeUniverseRedshift(0.75, 0.25, 0, 0.68, 1500)
```

[4]:
```
329759.843165775
```

Plot of the difference between apparent and absolute magnitudes, i.e., $m - M$, for a given redshift in a zero curvature universe.

[5]:
```
def RedshiftKZero(OmegaLambda,OmegaM,OmegaR,h,z):
    return numerical_integral(1/(x^2*sqrt(OmegaLambda+OmegaM/x^3+OmegaR/x^4)),1/
 ↪(1+z),1)[0]*(1+z)*3*10^8/h
def mM(OmegaLambda,OmegaM,OmegaR,h,z):
    return 5*log(RedshiftKZero(OmegaLambda,OmegaM,OmegaR,h,z))/log(10.)
```

[6]:
```
plot(lambda x:mM(0.76, 0.24, 0, 0.64,x),(x,0.004,1), axes_labels=[r'$z$',␣
 ↪r'$m-M$'],frame=True).show(figsize=5)
```