Aging, rheology, and overpopulated tails in sheared granular gases

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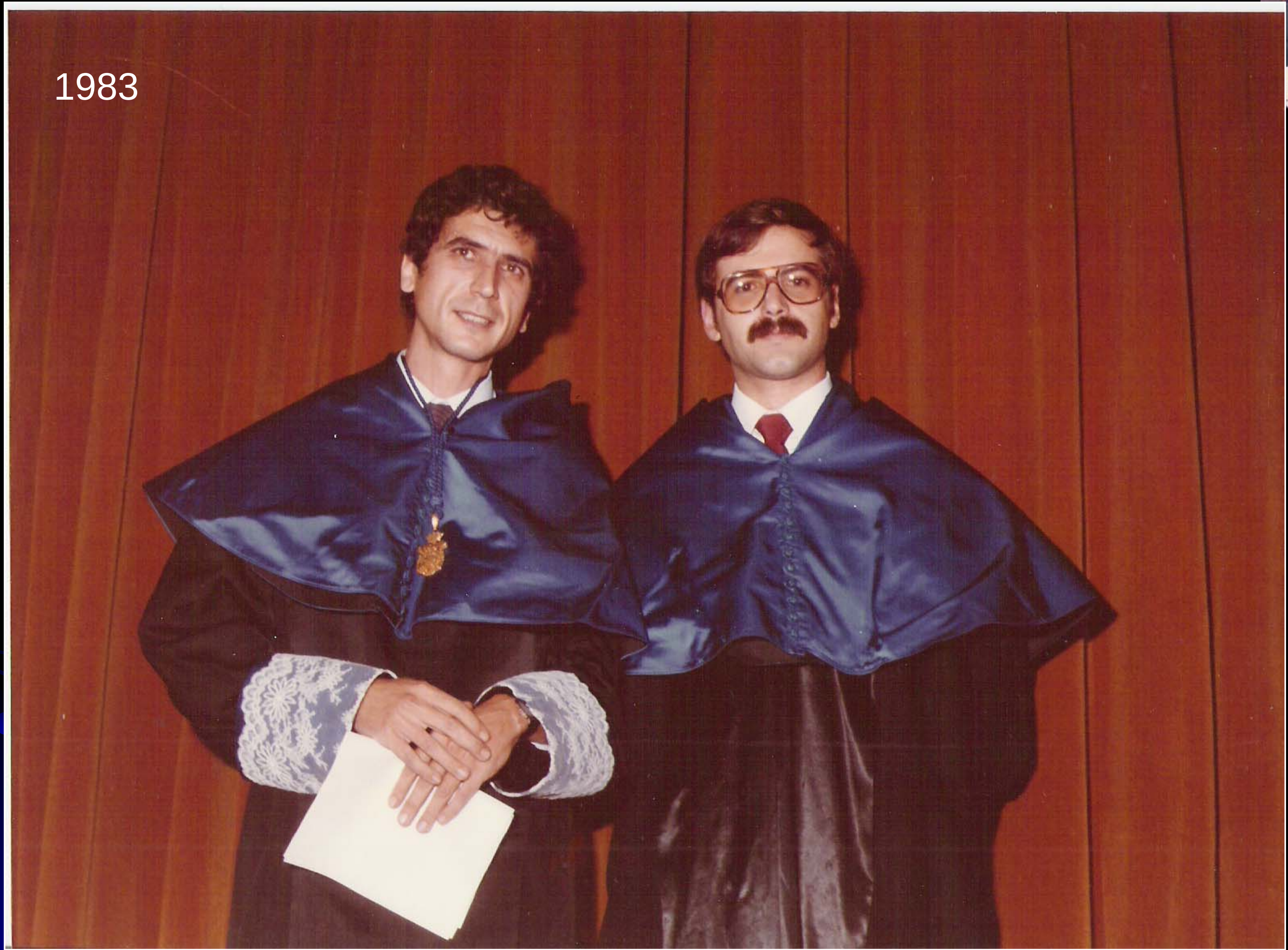

1983

Hydrodynamic description in normal gases

- Conservation equations (mass, momentum, and energy):

$$\underbrace{\partial_t y_i(\mathbf{r},t)}_{\text{Hydrodynamic fields}} + \nabla\cdot\underbrace{\mathbf{J}_i(\mathbf{r},t)}_{\text{Fluxes}} = 0$$

- Constitutive equations:

$$\mathbf{J}_i(\mathbf{r},t) = \mathcal{F}_i[\{y_j\}]$$

Closed set of equations

Hydrodynamic description in normal gases

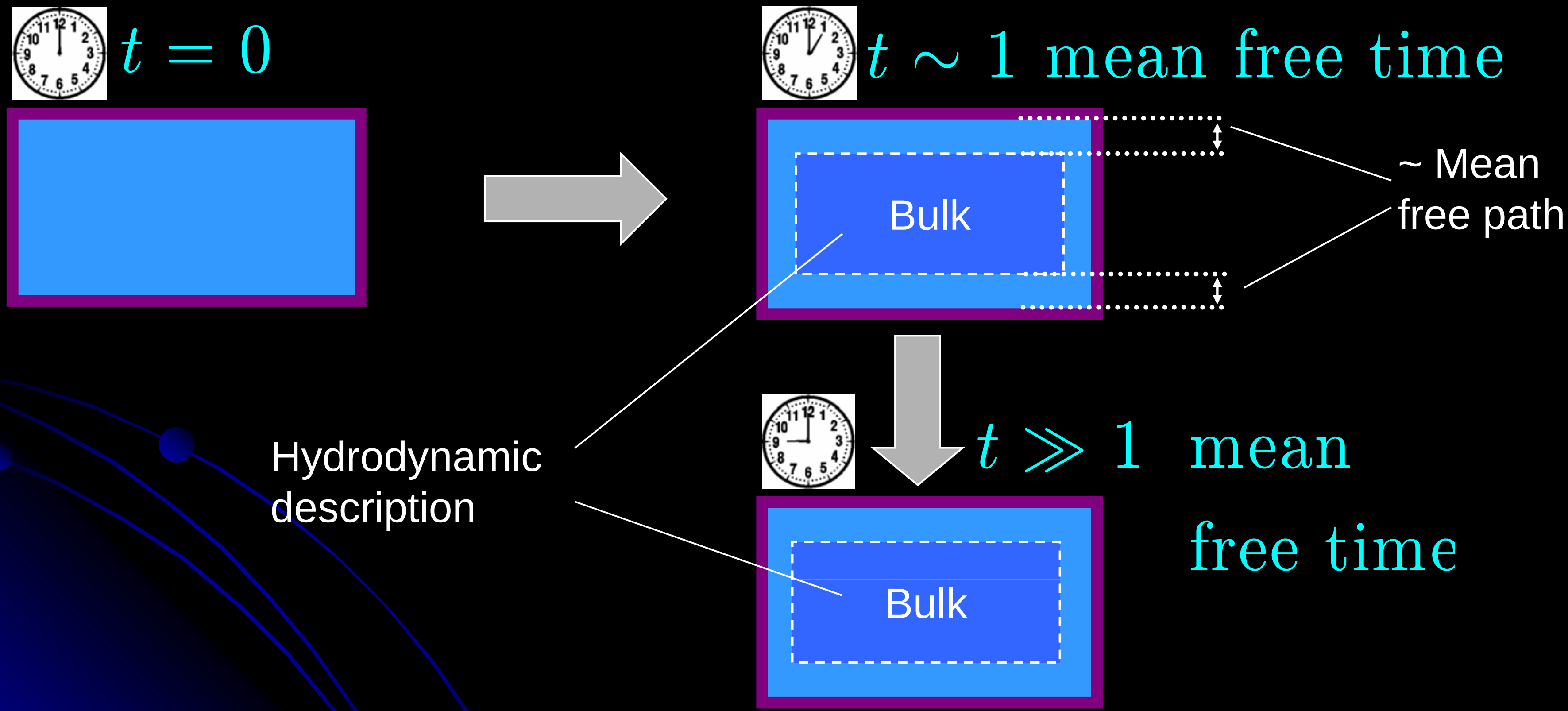
E.g., Navier-Stokes:

$$\mathbf{J}_i(\mathbf{r},t) = \mathbf{J}_i^{\mathrm{le}}(\mathbf{r},t) - \sum_j \lambda_j(\mathbf{r},t)\nabla y_j(\mathbf{r},t)$$

… but a hydrodynamic description is not restricted to the Navier-Stokes constitutive equations (non-Newtonian behavior, rheological properties, …)

$$\mathbf{J}_i(\mathbf{r},t) = \mathcal{F}_i[\{y_j\}]$$

"Aging" to hydrodynamics in normal gases



Low-density (<u>normal</u>) gases: the Boltzmann equation

$$\partial_t f + \mathbf{v} \cdot \nabla f = J[f, f]$$

$$\left.\begin{array}{l} f(\mathbf{v}, \mathbf{r}, 0) = f_0(\mathbf{v}, \mathbf{r}) \\ \text{boundary conditions} \end{array}\right\} \Rightarrow f(\mathbf{v}, \mathbf{r}, t) = \mathcal{F}[f_0; \mathbf{v}, \mathbf{r}, t]$$

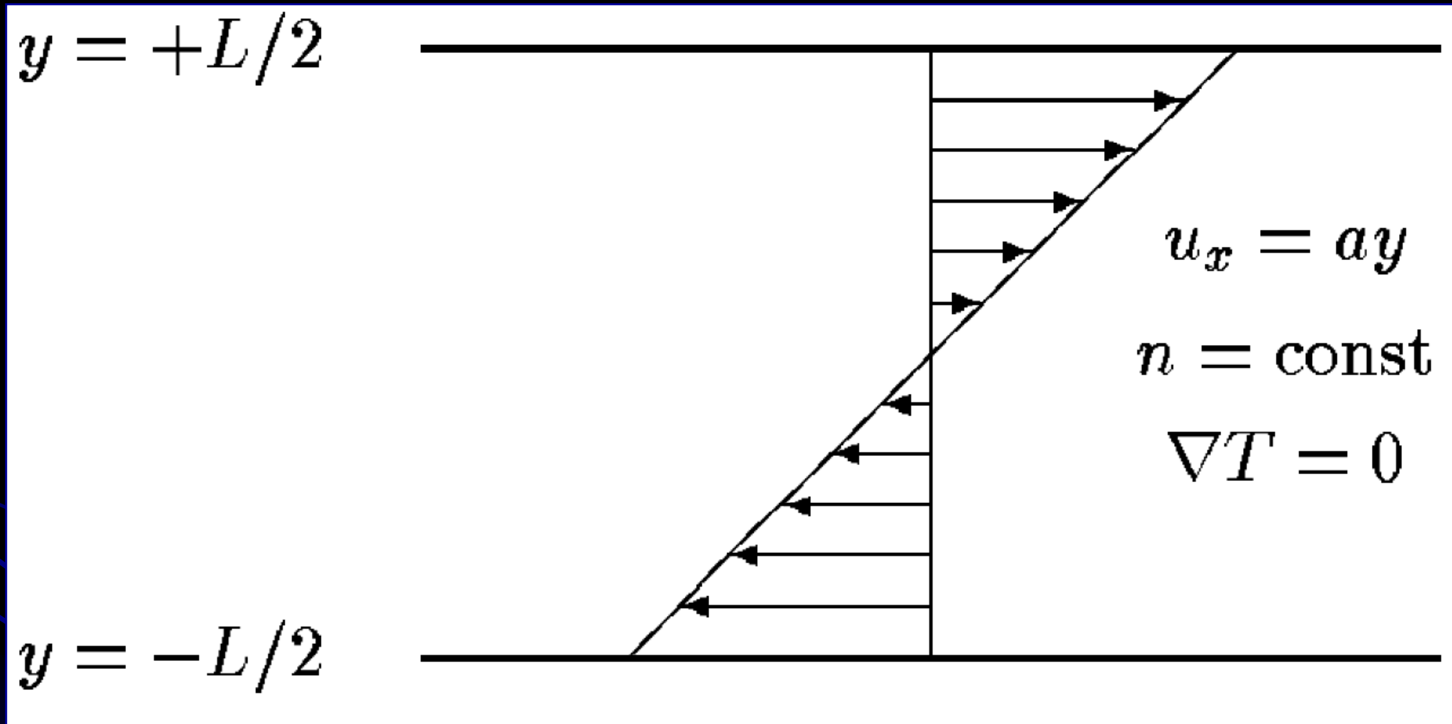
1. Kinetic stage $(t \sim 1 \text{ mean free time})$:

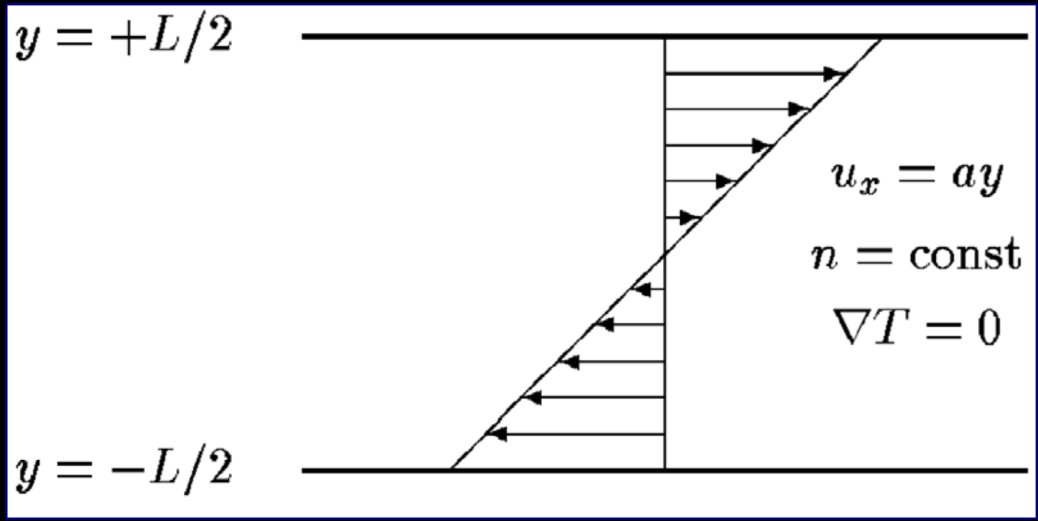
   Sensitive to the initial preparation

2. Hydrodynamic stage $(t \gg 1 \text{ mean free time}) \Rightarrow$

   "Normal" solution: $f(\mathbf{v}, \mathbf{r}, t) = \mathcal{F}[\{y_i\}; \mathbf{v}]$

Paradigmatic nonequilibrium state: Simple or Uniform Shear Flow





$$\partial_t T = \underbrace{-\frac{2}{3} a P_{xy}}_{\text{Viscous heating}} \Rightarrow T(t) \text{ monotonically increases}$$

Scaled shear rate: $a^* \equiv \dfrac{a}{\nu(T(t))}$ (decreases in time)

"Aging" to hydrodynamics

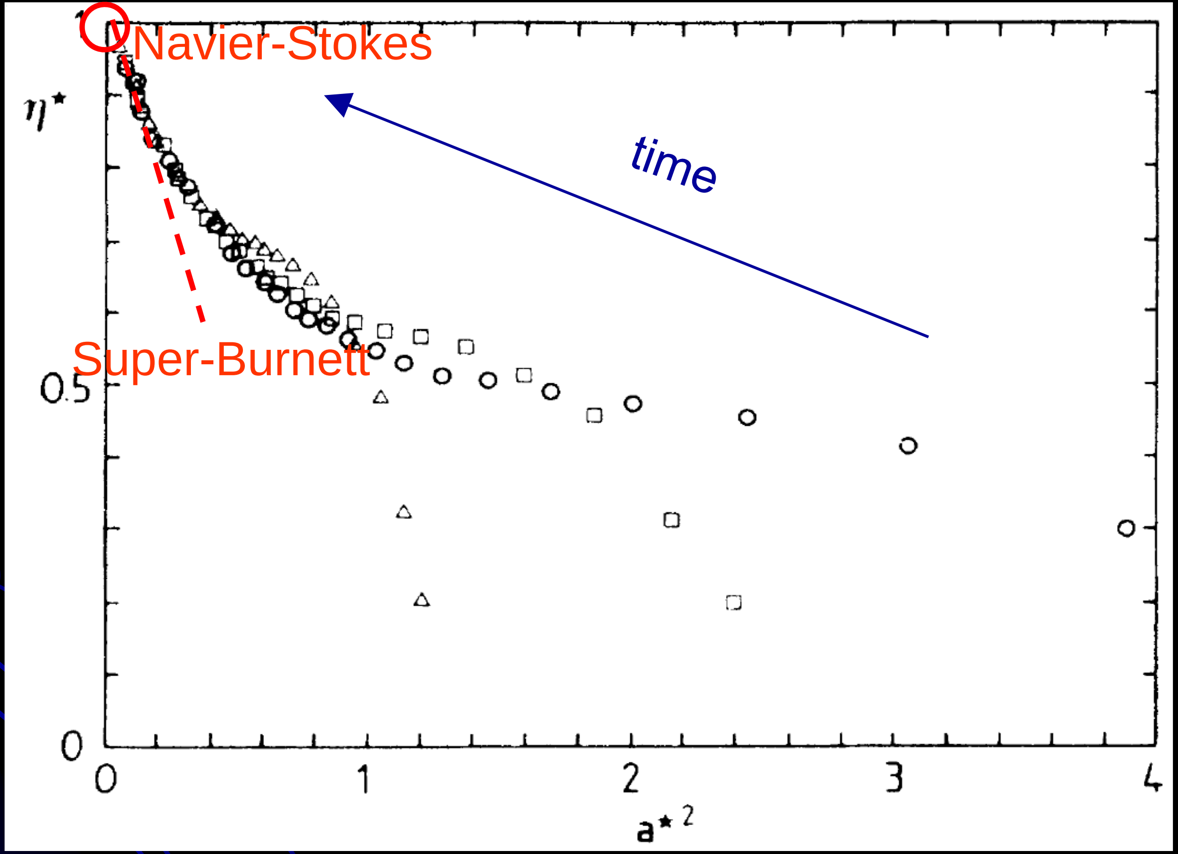
$$f(\mathbf{v},\mathbf{r},t) = \mathcal{F}[f_0;\mathbf{v},\mathbf{r},t] \to \mathcal{F}[\{y_i\};\mathbf{v}]$$

$$\text{USF} \Rightarrow f(\mathbf{v},\mathbf{r},t) \;\to\; n\left[\frac{m}{2T(t)}\right]^{3/2} f^*\left(\mathbf{C}(t);a^*(t)\right)$$

$$\mathbf{C}(t) \equiv \frac{\mathbf{v}-\mathbf{u}(\mathbf{r})}{\sqrt{2T(t)/m}}$$

$$P_{ij}(t) = \mathcal{F}_{ij}[f_0;t] \to nT(t)P^*_{ij}(a^*(t))$$

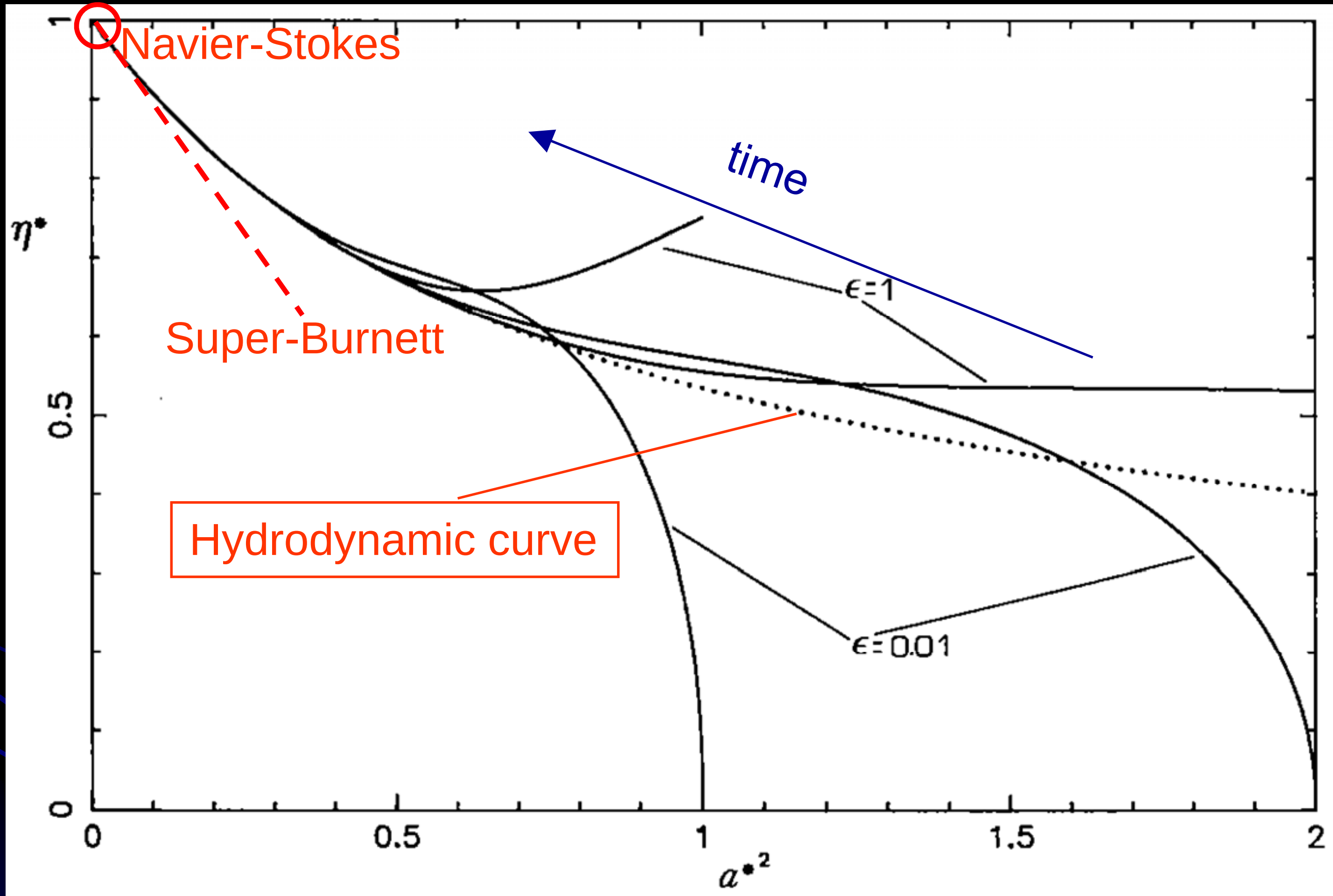
$$\eta(t) \equiv -\frac{P_{xy}(t)}{a} \to \frac{nT(t)}{\nu(T(t))} \underbrace{\eta^*(a^*(t))}_{\text{Scaled shear viscosity}}$$



DSMC simulations

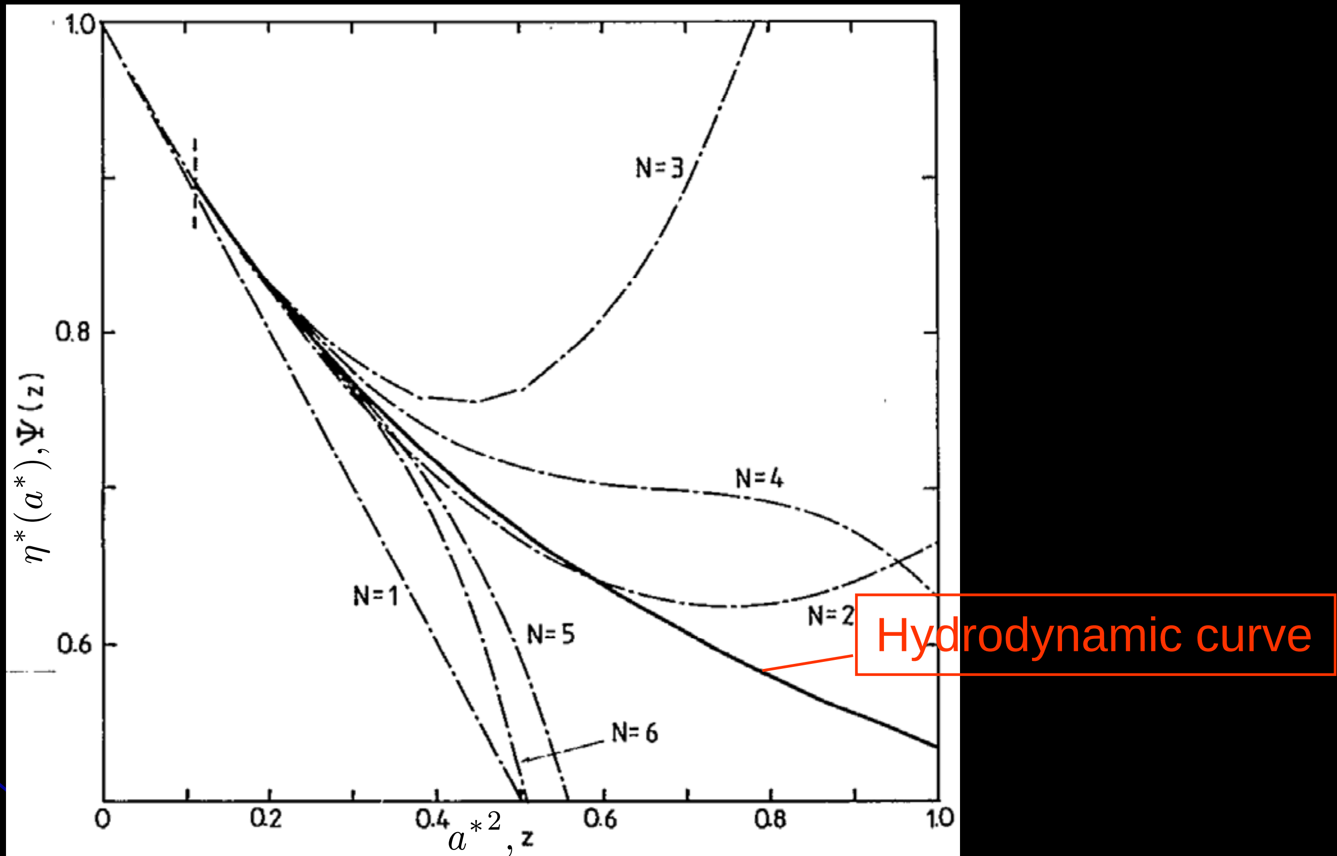
J. Gómez-Ordóñez, J. J. Brey, and A. S., Phys. Rev. A **39**, 3038 (1989)

● Kinetic model (BGK)



A. S. and J. J. Brey, Physica A **174**, 355 (1991)

- Divergence of the Chapman-Enskog expansion



A. S., J. J. Brey, and J. W. Dufty, Phys. Rev. Lett. **56**, 1571 (1986)

“Aging” to hydrodynamics in granular gases?

- Does the conventional aging scenario (short kinetic stage followed by slow hydrodynamic stage) still apply to normal gases externally *driven* (e.g., by a thermostat)?
- And to granular gases?
  - Energy is intrinsically not conserved!

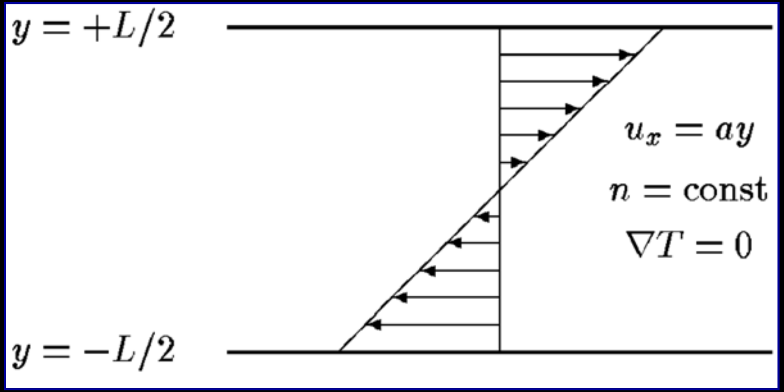
$$\left.\frac{\partial T(t)}{\partial t}\right|_{\rm coll} = -\boxed{\zeta(t)}T(t)$$

Cooling rate

L. P. Kadanoff, *Built upon sand: Theoretical ideas inspired by granular flows*, Rev. Mod. Phys. **71**, 435 (1999):

- Can a granular material be described by hydrodynamic equations, most specifically those equations which apply to an ordinary fluid?
- It seems to me that the answer is ‘‘no!’’.
- The study of collisions and flow in these materials requires new theoretical ideas beyond those in the standard statistical mechanics or hydrodynamics.
- One might even say that the study of granular materials gives one a chance to reinvent statistical mechanics in a new context.

Uniform shear flow of a granular gas



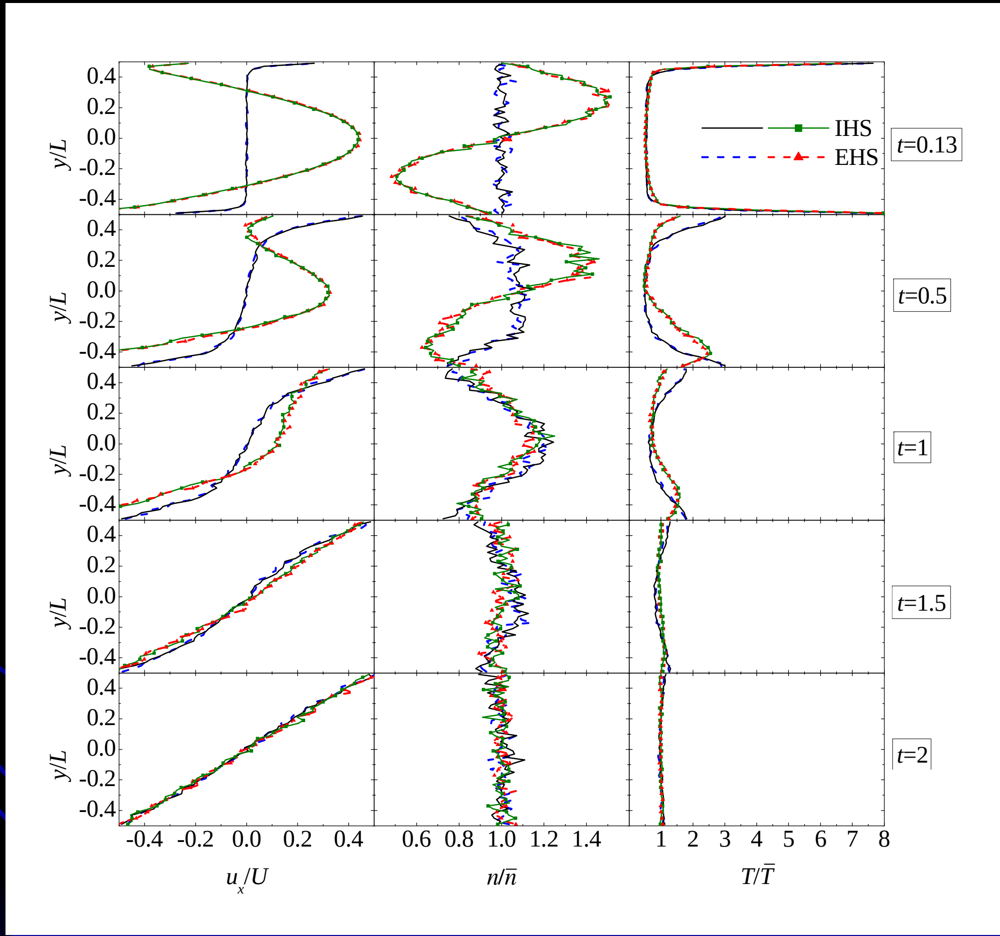
$$\partial_t T = \underbrace{-\frac{2}{3} a P_{xy}}_{\text{Viscous heating}} - \underbrace{\zeta T}_{\text{Inelastic cooling}} \Rightarrow T(t) \text{ reaches a stationary value}$$

$$\text{Scaled shear rate: } a^* \equiv \frac{a}{\nu(T(t))}$$

$$\text{Coefficient of restitution: } \alpha = \text{const}$$

DSMC simulations

$$\alpha = 0.9, a = 4 \Rightarrow T(t) \uparrow$$



“Homogeneization” of the hydrodynamic profiles

A. Astillero and A. S., Phys. Rev. E **72**, 031309 (2005)

Simple hydrodynamic solution of a *simple* kinetic model

$$\partial_t T = -\frac{2}{3n} a P_{xy}(t) - \zeta(T) T$$

$$\partial_t P_{xy} = -a P_{yy} - \frac{1+\alpha}{2} \nu(T) P_{xy} - \zeta(T) P_{xy}$$

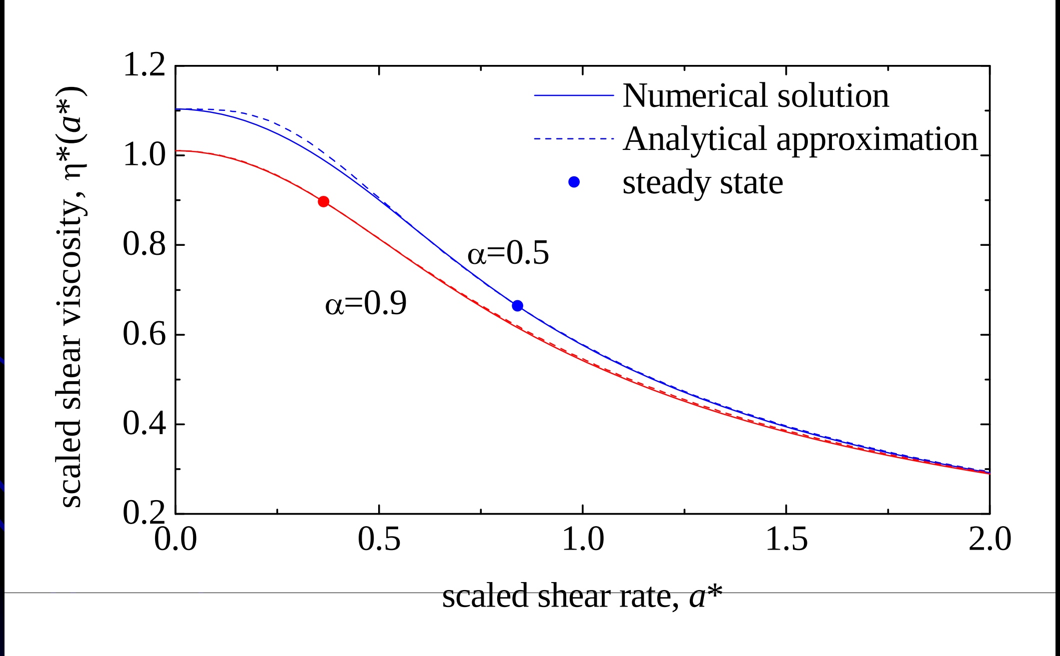
$$\partial_t P_{yy} = -\frac{1+\alpha}{2} \nu(T) \left(P_{yy} - nT\right) - \zeta(T) P_{yy}$$

$$\zeta(T) = \frac{5}{12}(1-\alpha^2)\nu(T), \quad \nu(T) \propto n T^{q}, \quad q = \frac{1}{2}$$

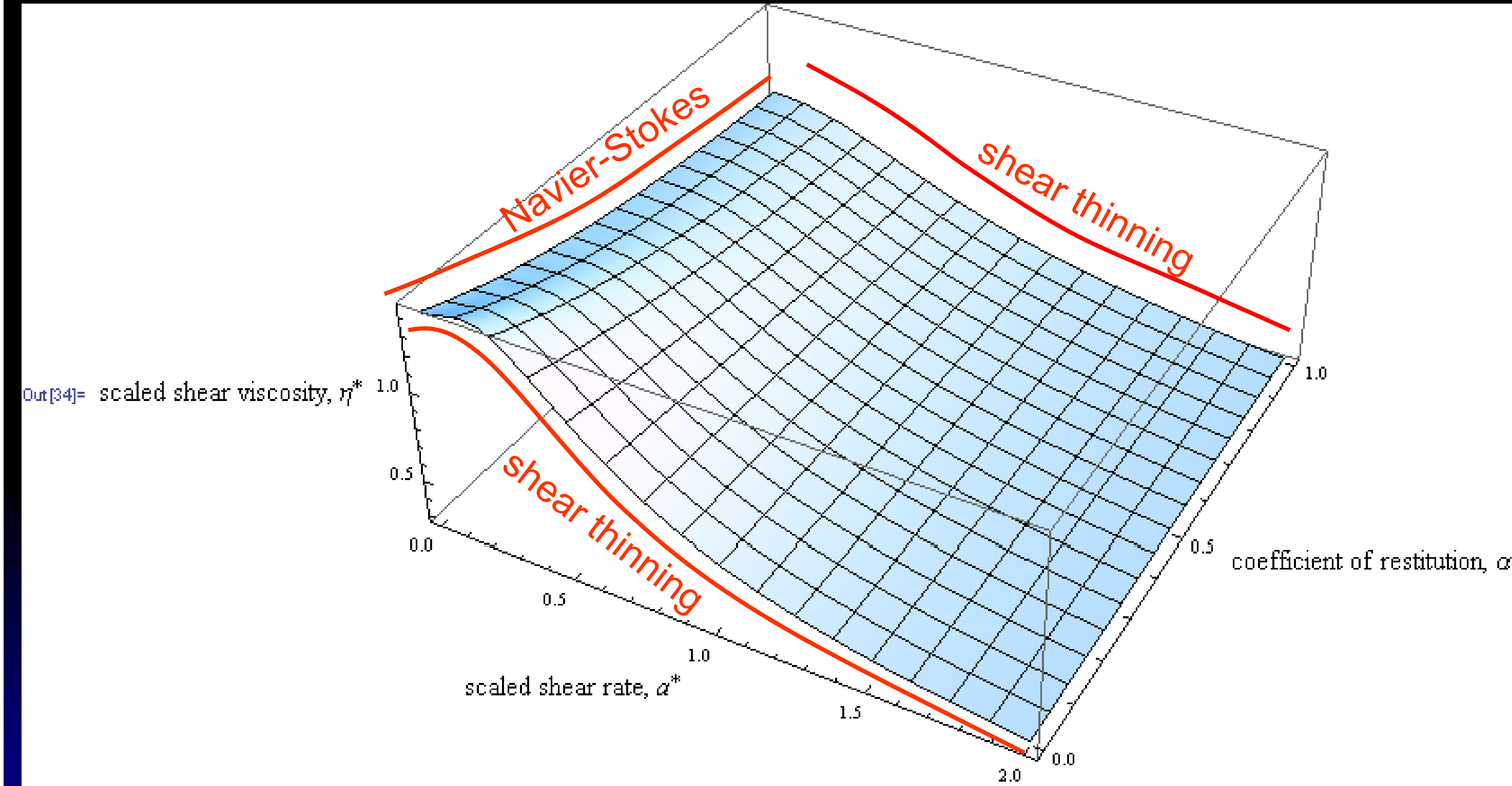
Simple hydrodynamic solution of a simple kinetic model

$$\eta^*(a^*,\alpha;q) = \eta^*(a^*,\alpha;0)\left[1-h(a^*,\alpha)q+\mathcal{O}(q^2)\right]$$

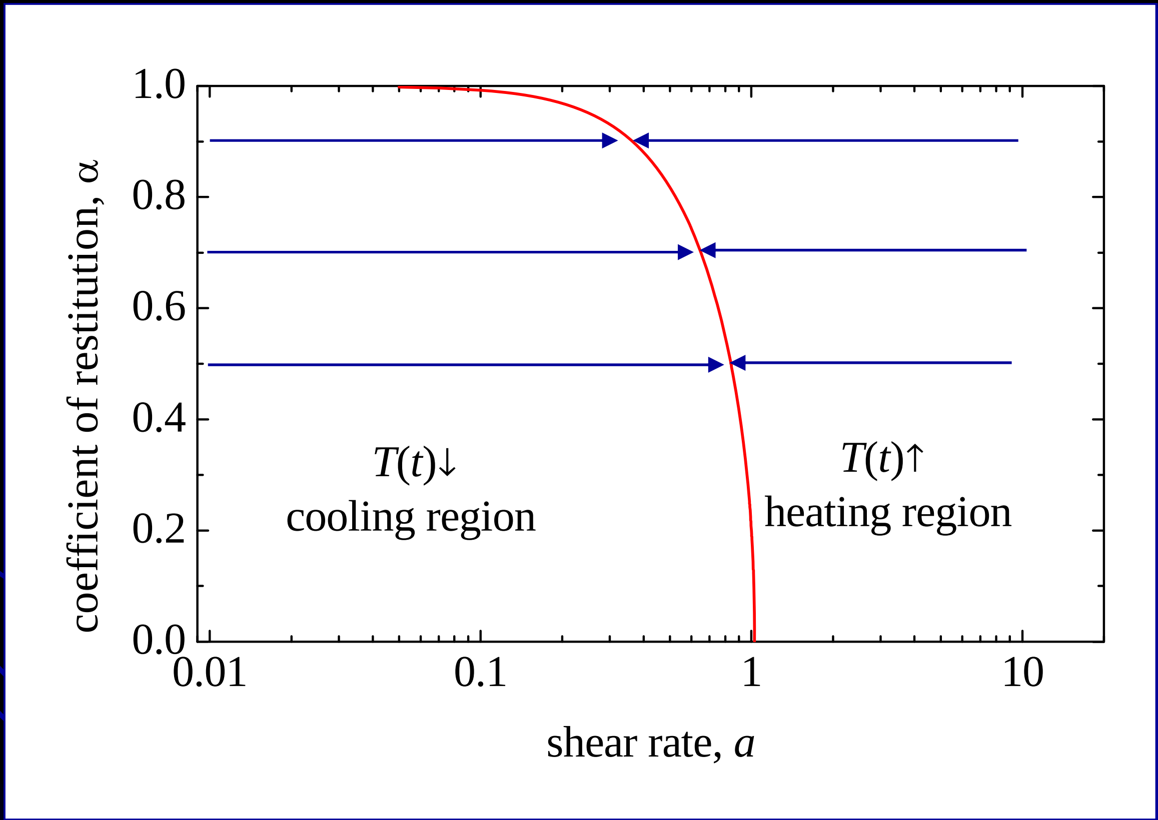
$$\simeq \frac{\eta^*(a^*,\alpha;0)}{1+h(a^*,\alpha)q}, \quad q=\frac{1}{2}$$



$$\eta^*(a^*, \alpha)$$


Out[34]=
scaled shear viscosity, η^*
1.0
0.5
0.0
0.5
1.0
1.5
2.0
scaled shear rate, a^*
0.0
0.5
1.0
coefficient of restitution, α
Navier-Stokes
shear thinning
shear thinning

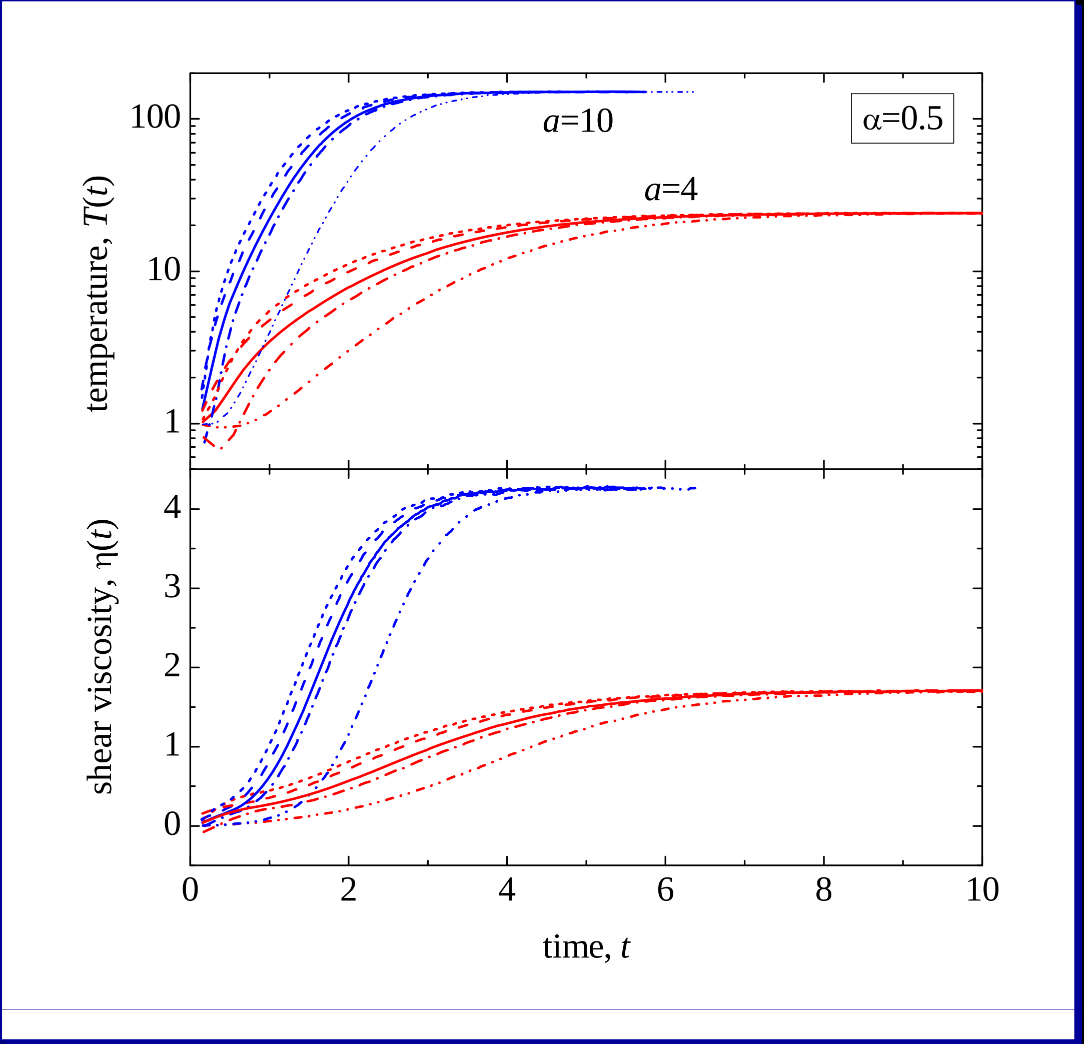
"Phase diagram": Competition between inelastic cooling and viscous heating



For each one of the 12 pairs (a^*,α), 5 different initial conditions

DSMC simulations

Relaxation toward the steady state

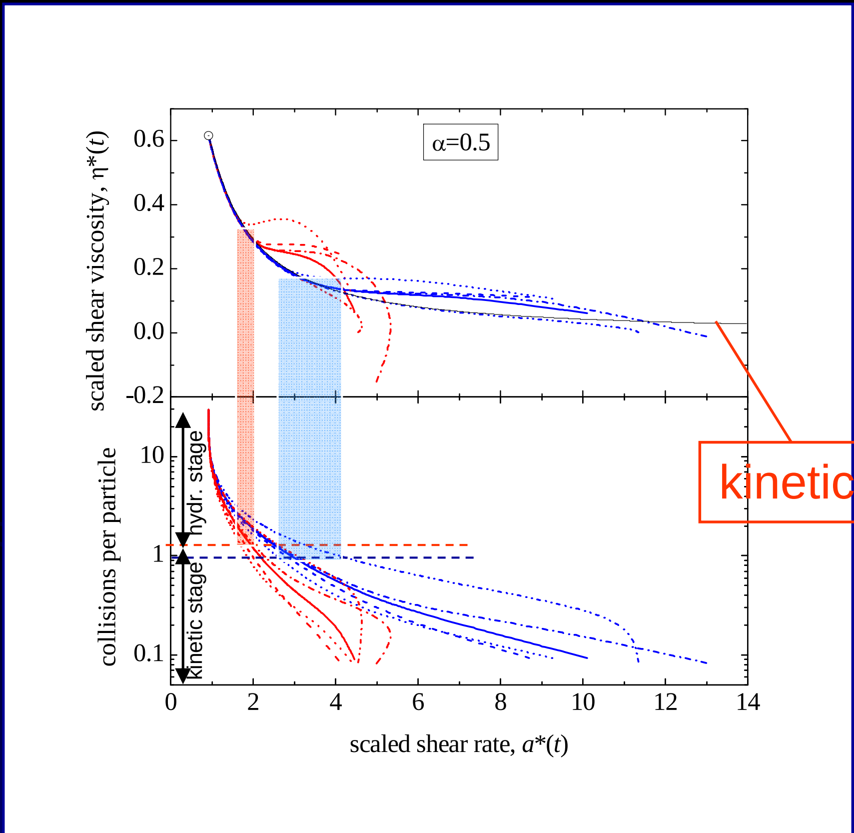


$$T(t) \to T_s(a, \alpha)$$
$$\eta(t) \to \eta_s(a, \alpha)$$

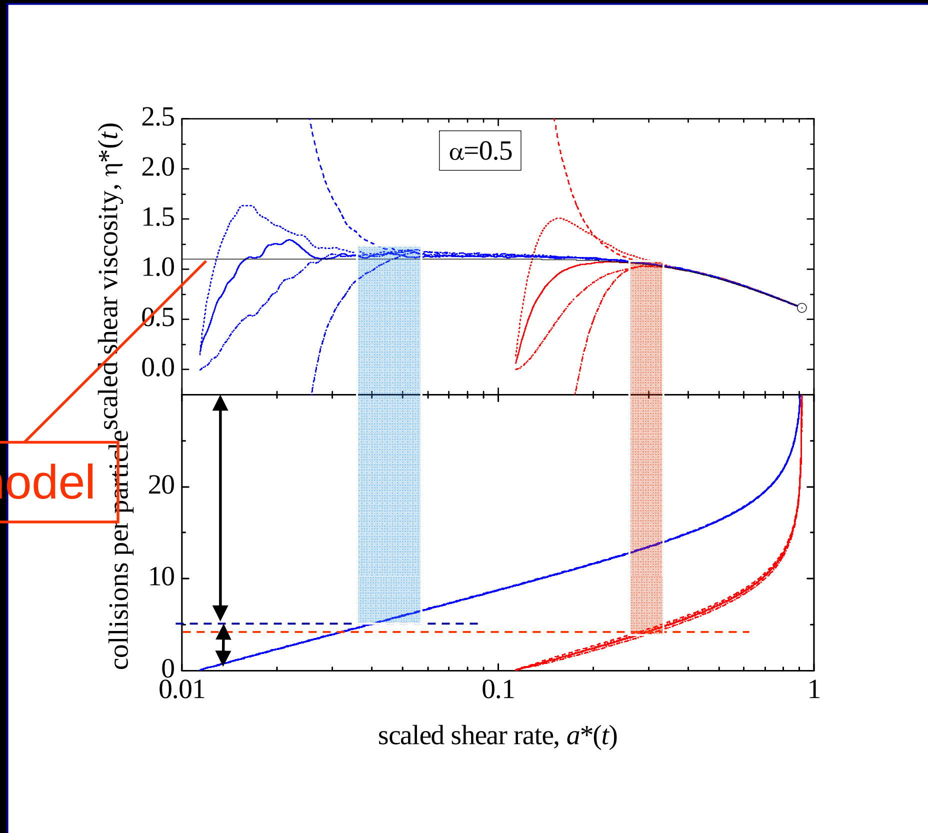
$$a^*(t) \to a^*_s(\alpha)$$
$$\eta^*(t) \to \eta^*_s(\alpha)$$

DSMC simulations

Unsteady hydrodynamic regime prior to the steady state?



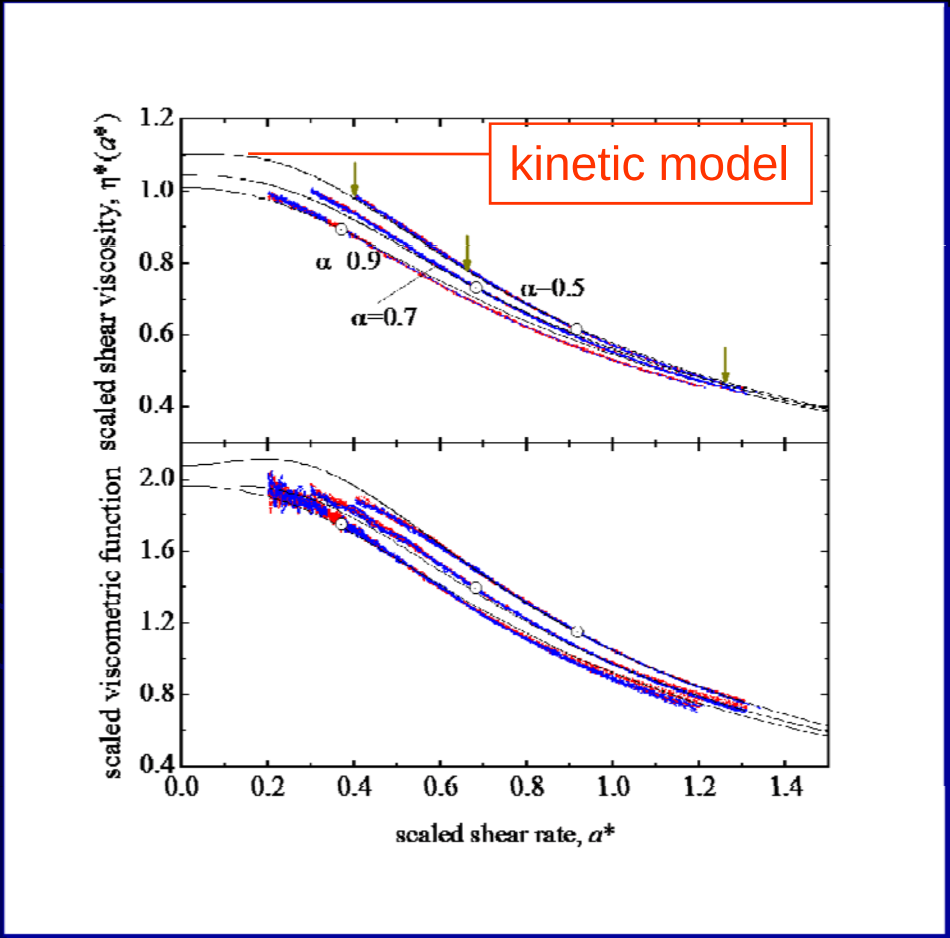
kinetic model



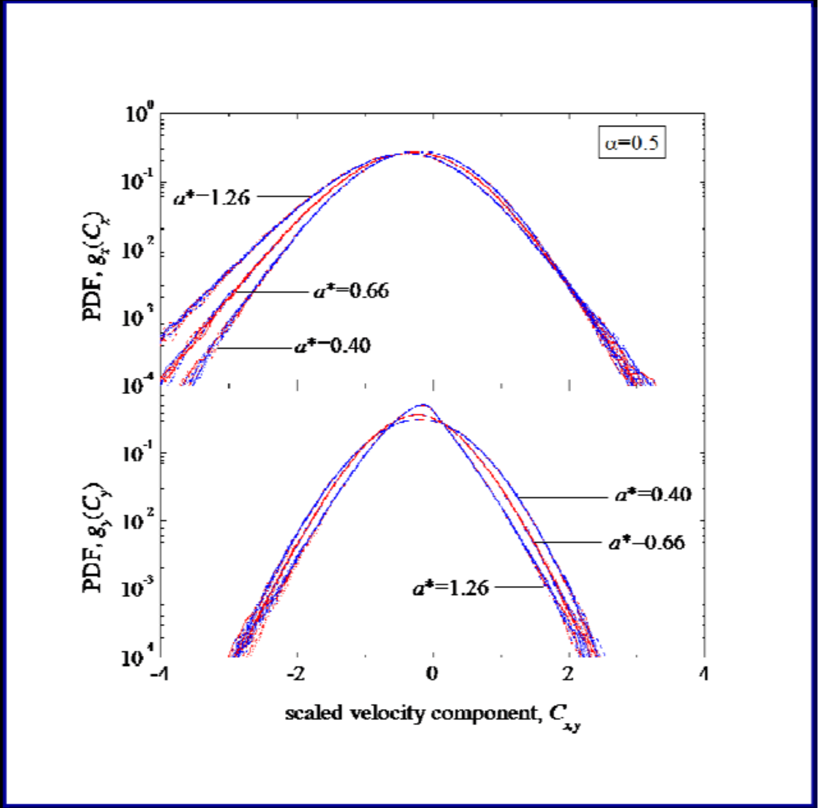
A. Astillero and A. Santos, Europhys. Lett. **78**, 24002 (2007)

DSMC simulations

Rheological quantities



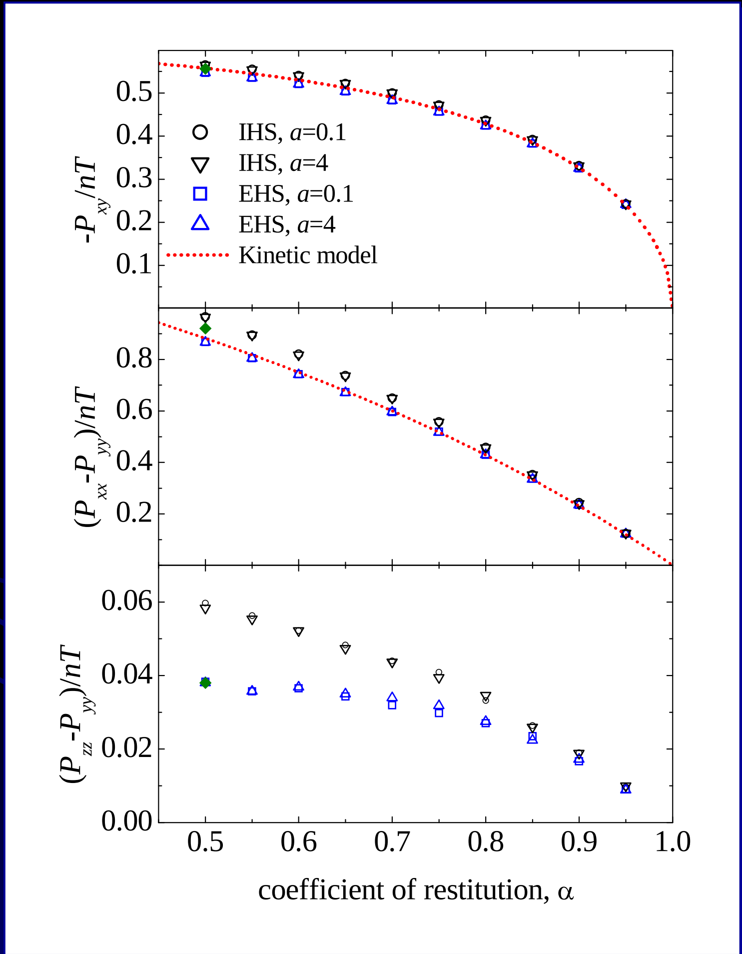
Velocity distribution



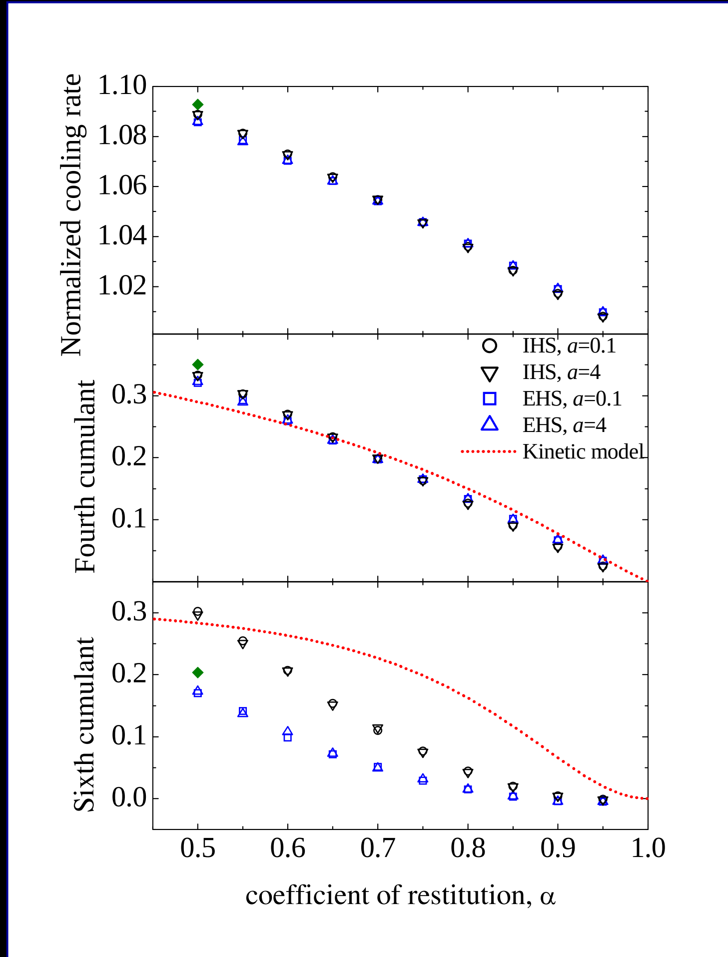
A. Astillero and A. S., Europhys. Lett. **78**, 24002 (2007)

DSMC simulations. Steady state

Rheological quantities



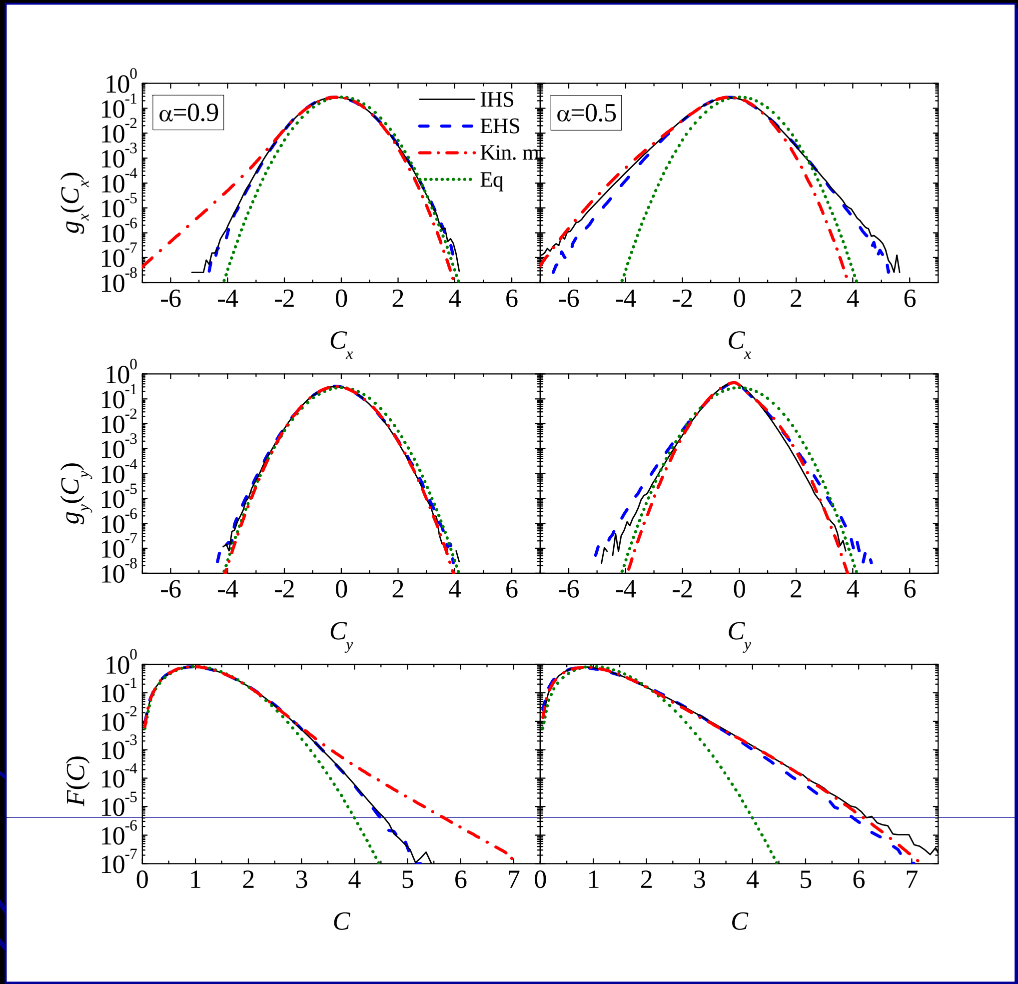
Higher velocity moments



A. Astillero and A. S., Phys. Rev. E **72**, 031309 (2005)

DSMC simulations. Steady state

Velocity distribution



A. Astillero and A. S., Phys. Rev. E **72**, 031309 (2005)

Conclusions

- The conventional scenario of aging to hydrodynamics seems to remain essentially valid for granular gases, even for non-Newtonian states and even when the time scale associated with inelastic cooling is shorter than the one associated with the irreversible fluxes.
- At a given value of α, the (scaled) nonlinear shear viscosity $\eta^*(a^*)$ moves on a certain rheological curve, the steady-state value $\eta^*_s=\eta^*(a^*_s)$ representing just one point.
- A good agreement is found by an (analytical) approximate solution to a simple kinetic model.
- The high-velocity tail of the velocity distribution function is consistent with an exponential overpopulation.
- The main nonequilibrium and transport properties of the true granular gas are satisfactorily mimicked by a *driven* gas of elastic particles.

¡Feliz cumpleaños, Javier (& Victoria)!