# COMMENTS

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## Comments on "A generalized BKW solution of the nonlinear Boltzmann equation with removal" [Phys. Fluids 27, 2599 (1984)]


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(Received 29 October 1985; accepted 12 December 1985)



It is shown that there is a direct relationship between the solutions of the Boltzmann equation for a homogeneous system of Maxwell molecules with and without particle removal.


Recently, Spiga[1] has found an exact, particular solution of the nonlinear Boltzmann equation for Maxwell molecules that includes a removal effect, making the total number of particles decrease in time. His special solution is a generalization of the formerly obtained solution in the absence of removal events (the so-called BKW mode[2]).

We show here that there exists a simple relationship between the solutions of both kinetic equations for arbitrary initial conditions. The Boltzmann equation for Maxwell molecules considered by Spiga[1] is

$$\frac{\partial}{\partial t} f(\mathbf{v},t) = -(C_S + C_R) n(t) f(\mathbf{v},t) + \int d\mathbf{v}_1 \int d\hat{\mathbf{n}}\, \alpha(\chi) f(\mathbf{v}',t) f(\mathbf{v}_1',t), \quad (1)$$

where $C_S = \int d\hat{\mathbf{n}}\, \alpha(\chi)$ and $C_R$ is the removal collision frequency. The evolution equation for the number density is

$$\frac{dn(t)}{dt} = -C_R [n(t)]^2, \quad (2)$$

whose solution is

$$n(t) = \frac{n_0}{(1 + n_0 C_R t)}, \quad (3)$$

where $n_0$ is the initial density.

Now, let us introduce the time-dependent parameter

$$\tau(t) \equiv \int_0^t ds \frac{n(s)}{n_0} = \frac{1}{n_0 C_R} \ln(1 + n_0 C_R t) \quad (4)$$

and the distribution function

$$\tilde{f}(\mathbf{v},\tau) \equiv [n_0/n(t)]\, f(\mathbf{v},t). \quad (5)$$

Thus, $\tilde{f}$ is proportional to the probability density function of the system, but now the role of time is played by $\tau$. Of course, when $C_R = 0$, $\tau = t$ and $\tilde{f} = f$. By substituting Eq. (5) into Eq. (1), we obtain

$$\frac{\partial}{\partial \tau} \tilde{f}(\mathbf{v},\tau) = -C_S n_0 \tilde{f}(\mathbf{v},\tau) + \int d\mathbf{v}_1 \int d\hat{\mathbf{n}} \times \alpha(\chi) \tilde{f}(\mathbf{v}',\tau) \tilde{f}(\mathbf{v}_1',\tau), \quad (6)$$

which is formally the usual Boltzmann equation (i.e., with conservation of the total number of particles). Thus, given an initial condition, the solution of Eq. (1) is closely related to the solution of Eq. (6). In particular, there exists a BKW-like solution to Eq. (1):

$$F(\mathbf{v},t) = \tilde{F}(\mathbf{v},\tau)/(1 + n_0 C_R t), \quad (7)$$

where $\tau$ is given by Eq. (4) and $\tilde{F}(\mathbf{v},\tau)$ is the well-known BKW mode.[2] The function $F$ is precisely the solution obtained in Ref. 1 by using a heuristic method.


### ACKNOWLEDGMENTS

The authors are very thankful to Professor M. H. Ernst for an enlightening discussion about this work.



[1]G. Spiga, Phys. Fluids 27, 2599 (1984).

[2]Extensive reviews are given in M. H. Ernst, Phys. Rep. 78, 1 (1981); J. Stat. Phys. 34, 1001 (1984).





0031-9171/86/051750-01$01.90