

Chapter 42

First-Order Contributions to the Partial Temperatures in Dilute Binary Granular Suspensions



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Abstract The Boltzmann kinetic equation is considered to evaluate the first-order contributions $T_i^{(1)}$ to the partial temperatures in binary granular suspensions at low density. The influence of the surrounding gas on the solid particles is modeled via a drag force proportional to the particle velocity plus a stochastic Langevin-like term. The Boltzmann equation is solved by means of the Chapman–Enskog expansion around the local version of the reference homogeneous base state. To first-order in spatial gradients, the coefficients $T_i^{(1)}$ are computed by considering the leading terms in a Sonine polynomial expansion. In addition, the influence of $T_i^{(1)}$ on the first-order contribution $\zeta^{(1)}$ to the cooling rate is also assessed. Our results show that the magnitude of both $T_i^{(1)}$ and $\zeta^{(1)}$ can be relevant for some values of the parameter space of the system.

42.1 Introduction

In the last years, the understanding of granular matter under rapid flow conditions has raised the interest of many researchers due not only to its practical applications but also due to the fact that the understanding of its properties is really an exciting challenge. At a more fundamental level, the description of polydisperse granular mixtures (namely, the gaseous state of a mixture of smooth hard spheres with inelastic collisions) has been focused on the modification of the kinetic theory of molecular gases to properly adapt it to the dissipative character of collisions among particles.

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Although the influence of inelasticity is reflected in all the transport coefficients, the main new feature (as compared with ordinary or molecular fluids) in the hydrodynamic equations is the presence of the so-called cooling rate ζ in the energy balance equation. The cooling rate (which vanishes for elastic collisions) measures the rate of energy dissipation due to inelastic collisions [1, 2].

One of the most intriguing and surprising effects of inelasticity in granular mixtures is the failure of energy equipartition in the homogeneous cooling state [2]. This means that the partial temperatures T_i of each component are different from the global granular temperature. The energy nonequipartition (which is only due to the inelastic character of collisions) has important effects in problems such thermal diffusion segregation. On the other hand, a new contribution to the breakdown of energy equipartition (additional to the one caused by the inelasticity in collisions) has been reported very recently [3]. Although considered in previous works of ordinary gases [4], this new contribution (which is associated with a nonzero first-order term $T_i^{(1)}$ in the expansion of the partial temperatures in powers of the gradients) had not been accounted in previous works of granular mixtures. Since $T_i^{(1)}$ is proportional to the divergence of the flow velocity \mathbf{U} (i.e., $T_i^{(1)} = \varpi_i \nabla \cdot \mathbf{U}$), the coefficient ϖ_i is also involved in the evaluation of the first-order contribution $\zeta^{(1)}$ to the cooling rate ζ .

Although the coefficients ϖ_i have been computed in the case of *dry* granular mixtures (namely, a granular mixture where the influence of the interstitial fluid on the dynamics of grains is neglected), we are not aware of a similar calculation for binary granular suspensions at low density. The objective of this paper is to evaluate the coefficients ϖ_i in granular suspensions where the effect of the surrounding gas is modeled by means of an effective external force [5] composed by two terms: (i) a viscous drag term that mimics the friction of grains with the surrounding fluid and (ii) a stochastic term representing random and uncorrelated collisions between grains and fluid molecules. Once the first-order $T_i^{(1)}$ contributions to the partial temperatures are evaluated, as a complementary goal, we will also assess the impact of $T_i^{(1)}$ on $\zeta^{(1)}$.

42.2 Boltzmann Kinetic Equation for Binary Granular Suspensions

We consider a granular binary mixture of spheres of masses m_i and diameters σ_i ($i = 1, 2$). Spheres are assumed to be completely smooth so that, the inelasticity of collisions is characterized by the constant (positive) coefficients of restitution $\alpha_{ij} \leq 1$. The solid particles are immersed in a viscous gas of viscosity η_g . In the low-density regime, the set of Boltzmann equations for the one-particle velocity distribution functions $f_i(\mathbf{r}, \mathbf{v}, t)$ of the component i reads [2]

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla f_i - \gamma_i \Delta \mathbf{U} \cdot \frac{\partial f_i}{\partial \mathbf{v}} - \gamma_i \frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{V} f_i - \frac{\gamma_i T_{\text{ex}}}{m_i} \frac{\partial^2 f_i}{\partial v^2} = \sum_{j=1}^2 J_{ij}[f_i, f_j]. \quad (42.1)$$

Here, $\Delta \mathbf{U} = \mathbf{U} - \mathbf{U}_g$, \mathbf{U} and \mathbf{U}_g being the mean flow velocities of the solid particles and the interstitial gas, respectively, $\mathbf{V} = \mathbf{v} - \mathbf{U}$ is the peculiar velocity, and $J_{ij}[f_i, f_j]$ is the Boltzmann collision operator [2].

At low Reynolds numbers, it has been assumed in Eq. (42.1) that the effect of the surrounding molecular gas on the solid particles is modeled by a drag force proportional to $\mathbf{v} - \mathbf{U}_g$ plus a stochastic Langevin force representing Gaussian white noise. While the drag force mimics the friction of grains with the interstitial gas, the stochastic force models the kinetic energy gain of solid particles due to the interaction with the background gas. In addition, γ_i is the drag or friction coefficient associated with the component i and T_{ex} is the temperature of the external gas. Note that the viscosity of the solvent $\eta_g \propto \sqrt{T_{\text{ex}}}$. More details of this kind of Langevin-like models can be found in Refs. [6, 7]. Here, for the sake of simplicity, we consider the coefficients γ_i to be scalars proportional to the viscosity η_g [5]. In this case, according to the results obtained in lattice-Boltzmann simulations [8], γ_i is a function of the partial volume fractions $\phi_i = \pi^{d/2} / (2^{d-1} d \Gamma(\frac{d}{2})) n_i \sigma_i^d$, n_i being the number density of the component i . Based on the restriction that in the dilute limit every particle is only subjected to its respective Stokes drag [8], $\gamma_i = \gamma_0 R_i$ where $\gamma_0 = (18\eta_g / \rho \sigma_{12}^2)$ and the dimensionless function $R_i = (\rho \sigma_{12}^2 / \rho_i \sigma_i^2) \phi_i$. Here, $\sigma_{12} = (\sigma_1 + \sigma_2) / 2$, $\rho = \sum_i \rho_i$ is the total mass density, and $\rho_i = m_i n_i$ is the mass density of the component i . Upon deriving the form of γ_i we have considered hard spheres ($d = 3$) and binary mixtures ($i = 1, 2$).

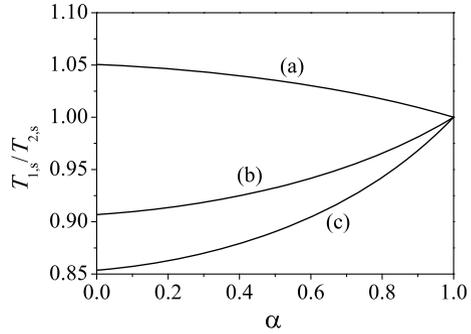
42.3 Homogeneous Steady States

Before analyzing inhomogeneous states, it is desirable to study first the homogeneous case. In this situation, the partial densities n_i are constant, the granular temperature $T = x_1 T_1 + x_2 T_2$ is spatially uniform, and with an appropriate selection of the reference frame, the mean flow velocities vanish ($\mathbf{U} = \mathbf{U}_g = \mathbf{0}$). Here, $x_i = n_i / n$ and $n = n_1 + n_2$ is the total number density. After a transient regime, the system is expected to reach a *steady* state and so, Eq. (42.1) becomes

$$-\gamma_i \frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{v} f_i - \frac{\gamma_i T_{\text{ex}}}{m_i} \frac{\partial^2 f_i}{\partial v^2} = \sum_{j=1}^2 J_{ij}[f_i, f_j]. \quad (42.2)$$

For elastic collisions ($\alpha_{ij} = 1$), the cooling rate vanishes and in the case that $\gamma_1 = \gamma_2 = \gamma$, Eq. (42.2) admits the Maxwellian solution with a common temperature $T_1 = T_2 = T$. For inelastic collisions ($\alpha_{ij} \neq 1$), $\zeta \neq 0$ and to date the solution of Eq. (42.2) is not known. Thus, one has to consider approximate forms for f_i . Here,

Fig. 42.1 Plot of the temperature ratio $T_{1,s}/T_{2,s}$ as a function of the common coefficient of restitution α for an equimolar mixture ($x_1 = 0.5$) of hard spheres ($d = 3$) with $\sigma_1/\sigma_2 = 1$, and $T_{\text{ex}}^* = 0.1$. Three different values of the mass ratio are considered: $m_1/m_2 = 0.5$ (a), $m_1/m_2 = 4$ (b), and $m_1/m_2 = 10$ (c)



for the sake of simplicity and to compute the first velocity moments of f_i , we will replace f_i by the Maxwellian distribution at the temperature T_i :

$$f_i(\mathbf{v}) \rightarrow f_{i,M}(\mathbf{v}) = n_i \left(\frac{m_i}{2\pi T_i} \right)^{d/2} \exp\left(-\frac{m_i v^2}{2T_i}\right), \quad (42.3)$$

where the partial temperatures T_i are defined as

$$T_i = \frac{m_i}{dn_i} \int d\mathbf{v} V^2 f_i(\mathbf{v}). \quad (42.4)$$

The (reduced) partial temperatures $\tau_{i,s} = T_{i,s}/T_s$ can be obtained by multiplying both sides of Eq. (42.2) by $m_i v^2$ and integrating over velocity. The result is $2\gamma_{i,s}^* (\tau_{i,s} - \theta_s^{-1}) + \zeta_{i,s}^* \tau_{i,s} = 0$, where the subscript s means that all the quantities are evaluated in the steady state. Here, $\gamma_{i,s}^* = \ell \gamma_{i,s}/v_{0s}$, $\theta_s = T_s/T_{\text{ex}}$, $\zeta_{i,s}^* = \ell \zeta_{i,s}/v_{0s}$, $v_{0s} = \sqrt{2T_s/\bar{m}}$ is the thermal speed, $\bar{m} = \sum_i m_i/2$, and $\ell = 1/n\sigma_{12}^{d-1}$ is the mean free path of elastic hard spheres. In the Maxwellian approximation (42.3), the (reduced) partial cooling rates $\zeta_{i,s}^*$ for the partial temperatures $T_{i,s}$ can be easily evaluated. Its explicit form can be found for instance in Eq. (5.51) of Ref. [2]. The temperature ratio $T_{1,s}/T_{2,s}$ is plotted in Fig. 42.1 versus the (common) coefficient of restitution $\alpha \equiv \alpha_{ij}$. As expected, failure of equipartition in the homogeneous steady state is presented when the collisions are inelastic ($\alpha \neq 1$). In addition, it is quite apparent that the extent of the energy violation is greater when the mass disparity is large.

42.4 First-Order Contributions to the Partial Temperatures

We now slightly perturb the homogeneous steady state by small spatial gradients. These gradients give rise to nonzero contributions to the mass, momentum, and heat fluxes. Here, we want to compute the first-order contributions to the partial temperatures. In order to achieve them, we have to solve the Boltzmann equation (42.1)

by means of the Chapman–Enskog method [9] adapted to dissipative dynamics. As usual, the Chapman–Enskog method assumes the existence of a *normal* or hydrodynamic solution where all the space and time dependence of the velocity distribution functions $f_i(\mathbf{v}, \mathbf{r}, t)$ occurs via a functional dependence on the hydrodynamic fields. For small spatial gradients, this functional dependence can be made local in space through an expansion of the distribution functions in powers of the spatial gradients: $f_i \rightarrow f_i^{(0)} + f_i^{(1)} + \dots$. Here, only terms up to first order in gradients will be retained (Navier–Stokes hydrodynamic order).

The first-order contributions $T_i^{(1)}$ to the partial temperatures are defined as

$$T_i^{(1)} = \frac{m_i}{dn_i} \int d\mathbf{v} V^2 f_i^{(1)}(\mathbf{V}). \quad (42.5)$$

Given that the procedure to determine $T_i^{(1)}$ is relatively large, we display here only the final results. More technical details can be found in Ref. [10]. As said in the Introduction, $T_i^{(1)}$ can be written as $T_i^{(1)} = \varpi_i \nabla \cdot \mathbf{U}$. In the leading Sonine approximation, the coefficients ϖ_i obey the set of algebraic equations

$$\begin{aligned} \sum_{j=1}^2 \left[\omega_{ij} + 2\gamma_j x_j (\tau_i + \theta \Delta_{\theta,i}) - 2\gamma_i \delta_{ij} + (T_i^{(0)} + T\theta \Delta_{\theta,i}) \xi_j \right] \varpi_j = \\ -\frac{2}{d} T\theta \Delta_{\theta,i} - T \sum_{j=1}^2 n_j \frac{\partial \lambda_1}{\partial n_j} \Delta_{\lambda_1,i}, \end{aligned} \quad (42.6)$$

where the coefficients ξ_i are defined by Eq. (25) of Ref. [10] while the collision frequencies ω_{ii} and ω_{ij} are given by Eqs. (26) and (27), respectively, of Ref. [10]. Moreover, the expressions of the derivatives $\Delta_{\theta,i}$, $\partial \lambda_1 / \partial n_j$, and $\Delta_{\lambda_1,i}$ are displayed in Ref. [10]. Upon deriving Eq. (42.6), use has been made of the identity $\zeta^{(1)} = \sum_i \xi_i \varpi_i \nabla \cdot \mathbf{U} \equiv \zeta_U \nabla \cdot \mathbf{U}$. The solution to the set of equations (42.6) provides the explicit forms of ϖ_1 and ϖ_2 . It is seen that $\varpi_2 = -(x_1/x_2)\varpi_1$, as the solubility conditions of the Chapman–Enskog method requires.

Figure 42.2 shows the α -dependence of $\varpi_1^* \equiv (n\sigma_{12}^2 v_0/T)\varpi_1$ and ζ_U for different systems. We observe that the influence of the inelasticity on both ϖ_1^* and ζ_U is important, specially for strong inelasticity. Thus, both quantities should be taken into account in the kinetic description of binary granular suspensions.

In summary, we have determined the first-order contributions $T_i^{(1)}$ to the partial temperatures in binary granular suspensions. The fact that $T_i^{(1)} \neq 0$ yields a *new* contribution (additional to the one caused by inelasticity in collisions) to the breakdown of energy equipartition. Since this contribution is proportional to the divergence of the flow velocity, it is involved then in the evaluation of the first-order contribution ζ_U to the cooling rate. Our results show that the magnitude of both coefficients $T_i^{(1)}$ and ζ_U can be significant in some regions of the parameter space of the system. This conclusion contrasts with the results obtained for dry granular mixtures [11, 12] where it was shown that both coefficients vanish in the low density limit.

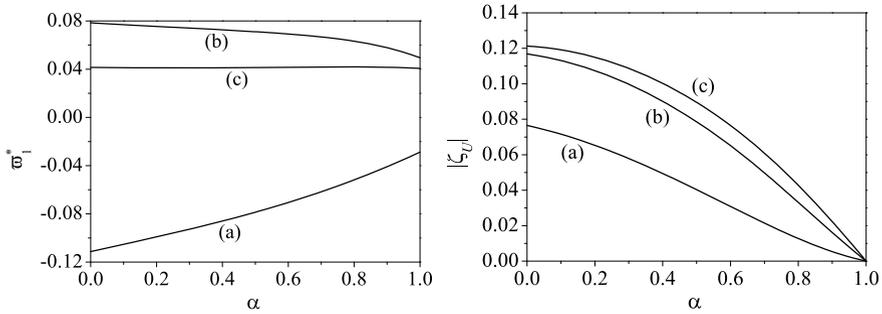


Fig. 4.2.2 Plot of the (reduced) coefficients ϖ_1^* and ζ_U as a function of the common coefficient of restitution α for an equimolar mixture ($x_1 = 0.5$) of hard spheres ($d = 3$) with $\sigma_1/\sigma_2 = 1$, and $T_{\text{ex}}^* = 0.1$. Three different values of the mass ratio are considered: $m_1/m_2 = 0.5$ (a), $m_1/m_2 = 4$ (b), and $m_1/m_2 = 10$ (c)

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