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In the droplet theory (in this theory the spin glass is an "adapted ferromagnet") the blocked probability distribution $P_R(q_R)$ should tend to the sum of two Dirac deltas approximately at $q_R \approx q_{EA}$ in the regime $1 \ll R \ll L$ (in order to avoid finite size effects).

We have studied the Binder parameter

$$g(R,t) \equiv \frac{3}{2} - \frac{\langle q_R^4 \rangle}{2\langle q_R^2 \rangle^2} \tag{14}$$

measured after t Monte Carlo steps. Using the dynamical finite size Ansatz we can write (for large R)

$$g(R,t) = f\Big(\big(R\xi(t)^{-1}\big)^{\delta} \Big) \ . \tag{15}$$

Our date follow very well this scaling Ansatz. The Binder function extrapolates to a value that is far away from the "droplet" value 1, the Binder cumulant of the sum of two Dirac deltas. We have checked this result for two different temperatures ($T = 0.35$ and 0.7).

Summarizing, we have shown different behaviours of different observables that can not be explained by the droplet theory, while the broken replica approach is able to predict qualitatively (and in a few cases even quantitatives) these behaviors. And so we believe that the broken phase of the three dimensional spin glass has the characteristics predicted by Mean Field.

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5 A Simple Equality

Using the properties of the Gaussian integrals is very easy to relate the link energy with the $q-q$ correlation function at distance 1

$$E_{\rm link} = -\beta(1 - C(1)) \ , \tag{10}$$

where $E_{\rm link}$ is the energy per link and $C(x)$ is the $q-q$ correlation function of the fully equilibrated system (i.e. taking into account all the possible ergodic components).

The value of the energy per link is very well fitted (as a function of the Monte Carlo time), and so we have a accurate value for $C(1)$, by the form

$$E^{\infty}_{\rm link} + At^{-\Delta(T)} \ . \tag{11}$$

The exponent $\Delta(T)$ is large, i.e. we find $\Delta(T) \simeq 0.44T$, so that it is easy to extrapolate the value of the energy to infinite time. Using the results of reference [14] (by computing the interface energy using SRSB theory) we expect that $\Delta(T) = 2.5\lambda(T)$, in a very good agree with our data.

If the replica symmetry is broken then our $q-q$ correlation function G musts differ from the "equilibrium" correlation function $C(x)$ because $G(x)$ only contains two ergodic components. The equality

$$C(1) = G(1) \tag{12}$$

should be violated if the replica symmetry is broken. This happens in the Sherrington-Kirkpatrick model in the spin glass phase where the equality $E = -\frac{\beta}{2}(1 - q^2_{EA})$ does not hold.

While we find that the equality (12) is good satisfied (with less that a relative 1% error) in the paramagnetic region and at the critical temperature, it is violated below T_c; at $T = 0.35$ we find $C(1) = 0.802 \pm 0.001$ and $G(1) = 0.67 \pm 0.01$, at $T = 0.7$ we obtain $C(1) = 0.612 \pm .001$ and $G(1) = 0.56 \pm 0.01$. The broken of the equality (12) shows the existence of different ergodic components, and then the overlap correlation function depends on the choice of the component, in agreement with one of the prediction of the SRSB theory.

6 Box Overlaps

In this last section we will analyze the local overlap in a box of side R,

$$q_R(x) \equiv R^{-D} \sum_y \sigma_{x+y}\tau_{x+y} \ , \tag{13}$$

where y is an integer vector which takes all the R^D values compatible with the conditions $0 \leq y_\nu < R$. We calculate the probability distribution $P_R(q_R)$ of the local box overlap.

In the mean field SRSB limit the function $P_R(q_R)$ is Gaussian, but in finite (not too large) dimensions it is reasonable to expect strong deviations from the previous Gaussian behaviour.

let $L \to \infty$ first we can use this approach to study the equilibrium value of the correlation function with the constraint of having zero overlap.

In order to do that we consider the time dependent equal time correlation function at time t

$$G(x,t) = V^{-1} \overline{\sum_i \langle \sigma_{i+x} \tau_{i+x} \ \sigma_i \tau_i \rangle_t} \ , \tag{7}$$

where the average is done at time t, i.e. after t Monte Carlo cycles after the random start. We find that for large times t the correlation function $G(x,t)$ is essentially different from zero for distances not too larger than a dynamic correlation lengthy $\xi(t)$ which increases (and maybe diverges) with time. Our numerical data are well represented with the functional form

$$G(x,t) = \frac{A(T)}{x^\alpha} \ \exp\Big\{ -\big(\frac{x}{\xi(T,t)}\big)^\delta \Big\} \ , \tag{8}$$

where we have defined $\xi(T,t) \equiv B(T) \ t^{\lambda(T)}$. In the whole range of distances $1 \leq x \leq 8$ for Monte Carlo times which range from 10^2 to 10^6 full lattice sweeps and a large range of temperatures $T < T_c$ (we have done measurements at different temperatures, down to $T_{min} \simeq .3 T_c$) we get good fits. The exponents α and δ are weakly dependent on T. For example at $T = 0.70$ we get the best values $\alpha = 0.50 \pm 0.02$ and $\delta = 1.48 \pm 0.02$. The correlation length exponent $\lambda(T)$ is approximately given by $0.16T$. Such power law growth of the correlation length was already observed by Rieger [13]. In order to study the limit $t \to \infty$ in a safe way it is even better to avoid global fits and to fit the data at fixed distance x as

$$G(x,t) = G(x,\infty) \ \exp\Big\{ -A(x) \ t^{-\lambda(T)} \Big\} \ . \tag{9}$$

In this way the extrapolation to infinite time (with the self-implemented constraint of $q = 0$ always satisfied) is performed in a very safe way. We have checked that the extrapolated correlators $G(x,\infty)$ (computed at $T = 0.7$) as a function of the distance follow a very good straight line in double logarithmic scale for x up to 5 (where the statistical error becomes large).

We have also computed the same quantities by using a different temperature schedule. In this second numerical experiment we slowly cool down the system from $T = 1.5 > T_c$ to the final temperature. To perform the cooling we use a number of steps proportional to t, the waiting time we want to look the correlation function at. After that the system evolves at the fixed temperature of interest T for t more time steps before measurement. In this way one can obtain a much better equilibration. As matter of principle in this case one does not expect a pure power law but a combination of different powers generated by different temperature contributions. However a fit similar to the previous one (9) works very well with a slightly large value of of λ. The data obtained with the two techniques behave in a very similar way. The $t = \infty$ data are well described by a power decay $x^{-\alpha}$ with $\alpha = 0.50 \pm .03$, as predicted by the replica theory and in variance with the droplet model predictions.

we have that in mean field

$$\overline{\langle q^k \rangle_J \langle q^m \rangle_J} \;=\; \frac{2}{3}\overline{\langle q^k \rangle_J}\;\;\overline{\langle q^m \rangle_J} \;+\; \frac{1}{3}\overline{\langle q^{k+m} \rangle_J} \;, \tag{4}$$

where by the overline we indicate the average over the quenched noise. We have verified that in the low T region this equality is very well satisfied. For example for $k = 2$ and $m = 2$ at $T = 0.7$ and L ranging from 4 to 10 the ratio of the l.h.s. to the r.h.s. of (4) is equal to 1.0 with an error never larger than 0.1. It is remarkable that Francesco Guerra has been able to prove [10] that (4) has to be exactly satisfied in mean field models with quenched disorder, and that the proof can also be generalized to systems defined in a finite number of dimensions: it will be important to understand in better detail how significant these relations are, and how far they can lead it on the route of detecting ultrametric like behaviors [11].

Strictly speaking the non-triviality of the function $P(q)$ is not in violent contradiction with the droplet model. In the framework of the droplet approach it is always possible to suppose that states where domains that take a finite part of the whole system are reversed have a finite probability. This hypothesis is however rather unnatural and it is definitely wrong in the Migdal-Kadanoff approximation. Moreover we have already seen that the ability of the SRSB theory to predict quantitatively the fluctuations of the function $P(q)$ is remarkable.

4 Time Dependent Correlation Functions

We will now falsify the possibility discussed in the last paragraph by considering the $q-q$ correlation functions restricted to those pairs of configuration which have a small value of q. The analysis of such correlation functions, together with the non-triviality of the $P(q)$, will constitute an ultimate test of the failure of the droplet model.

More precisely we consider a system of side L and we define the relevant correlation function as

$$C(x, L) = V^{-1} \overline{\langle \sum_i \sigma_{i+x}\tau_{i+x}\;\sigma_i\tau_i \rangle} \;, \tag{5}$$

where the brackets indicate the thermal average. The droplet model predicts that $C(x, \infty)$ goes to the constant value q_{EA}^2 for large x. In the SRSB approach

$$C(x, \infty) \propto |x|^{-\lambda} \;, \tag{6}$$

where λ in an appropriate exponent which has been computed in less than 6 dimensions for the $q = 0$ correlation functions [12].

We have studied this problem by considering large systems, with $L = 64$. We have run numerical simulations starting from two random configurations selected independently (for 4 realizations of the quenched couplings). We have verified that q^2 stays small in the whole run so that the difference in the initial configurations, for not too large times, enforces the condition $q \approx 0$. Eventually in a finite system global equilibrium will be reached and q will become of order 1. However if we

Mainly thanks to the use of large computer resources (we have mainly used the APE parallel computer [8], which turns out to be very effective for this kind of problems [6]: we flip about $2 \cdot 10^8$ spins per second on the *tower* version of the machine) and of the *tempering* (an annealing-like improved Monte Carlo technique introduced in [9]) we have been able to study systems of larger sizes than before (up to 14^3). We were able to bring the samples to thermal equilibrium quite deep in the cold phase. We will see that this information is complemented by our dynamical study, where we work on time scales on which we can equilibrate the system on distances up to order 6. This gives a good control over the fractal geometry of the typical excitations and of their boundaries. This is what we need in order to distinguish between SRSB theory and Migdal-Kadanoff droplets.

3 $P(q)$ and its Momenta

The first crucial comment is that some features of the shape of the function $P(q)$ are (within our statistical precision) size independent. For instance, the non-zero value of the *plateau* at low q values, down to $q = 0$, turns out to be size-independent. Also at $T = 0.7$ the Binder cumulant of q is practically independent of the lattice size and it is equal to $0.85 \pm .01$. This means that the system has a non-trivial structure of equilibrium states with a continuous distribution of the allowed overlap values. By using our measurements of equal time correlation functions we will argue in the following that such states cannot be described by the droplet approach, while they have all the features predicted by the SRSB approach.

In this note we do not answer a very important question, i.e. if in the infinite volume limit a low-temperature phase characterized by the existence of a non zero order parameter q_{EA} exists. On the lattice volumes we are able to investigate the high q peak of the $P(q)$ is very slowly shifting toward lower q values, even if, as we already said, the shape of the $P(q)$ does not change. The extrapolation to the infinite volume limit looks in this case a very delicate issue, and many potential systematic errors (even in the definition of the finite volume q_{EA}) are involved. Here we will not address in detail this point, and assume that we are working in conditions where the system is effectively frozen to a phase with a non-zero value of q_{EA}. A possible scenario [4] of a correlation length diverging exponentially for $T \to 0$ or of a Kosterlitz-Thouless like transition would be compatible with this approach, since on our finite lattice we would be measuring properties of a frozen system. It is also important to note that this ambiguity only concerns the behavior of the high q peak of the $P(q)$ (which could tend to $q = 0$ on very large lattices), while on the contrary the $P(q)$ for small q values is non-trivial and does not depend on the lattice size.

The agreement with mean field theory becomes quantitative if we study sample to sample fluctuations. Mean field theory tells us how much the function $P_J(q)$ for a given realization of the quenched disorder differs from the average. For example if we consider

$$\langle q^k \rangle_J \equiv \int dq \; P_J(q) \; q^k \; , \tag{3}$$

ref. [6]). We will start by showing that the probability distribution of the overlap among two systems at equilibrium, $P(q)$, has a non-trivial structure. $P(q \simeq 0)$ is different from zero, and its shape does not depend on the volume size. We will analyze (following a suggestion contained in the third reference of [7]) sample to sample fluctuations of the spin-glass susceptibility, and find out that they are incompatible with the droplet model, while their size is very well explained (even in a quantitative manner) by SRSB theory. In order to show that the structure of the different equilibrium states is not compatible with a droplet structure we will compute and analyze equal time correlation functions. From this analysis we deduce the existence of many equilibrium states that cannot be described by a droplet like structure. We will also show that even at a quantitative level SRSB theory explains very well the numerical data.

Further evidence about the inadequacy of the droplet model to describe the $3d$ spin glasses and support to a SRSB mechanism will be provided by analyzing the distribution of overlaps of boxes of side R, and by discussing the behavior of the box overlap Binder parameter.

2 The Numerical Simulation

The model we mainly consider is defined by the simple Edwards-Anderson Hamiltonian on a $3d$ simple cubic lattice

$$H \equiv -\sum_{\{i,k\}} \sigma_i J_{i,k} \sigma_j \ , \tag{1}$$

where the sum runs over nearest neighbor couples of sites. The quenched disordered couplings J are distributed according to a Gaussian law. A study of the overlap susceptibility and of the Binder cumulants shows that (under the a-priori assumption about the existence of a phase transition at a non-zero temperature with a power law divergence of the correlation length) the transition is located at $T \approx 1$. In order to check universality of our results we have also studied a model [6] with integer $J = \pm 1$ variables, where each spin is coupled with equal strength to 26 neighboring sites (all the ones contained in a cube of 3^3 sites). The results we discuss here are confirmed by our findings about this second model.

We have used an isotropic lattice of linear size L, and we have computed the probability distribution $P_J(q)$ of the overlap

$$q \equiv V^{-1} \sum_i \sigma_i \tau_i \tag{2}$$

among two thermalized configurations σ and τ in a box of volume $V = L^3$. We have studied the behavior of the function $P(q)$ averaged over a large number of realizations of the quenched disordered couplings J (i.e. the average over the J random variables of $P_J(q)$). We have used a maximum of 2560 samples for the smallest lattice sizes and a minimum of 512 samples for the largest sizes. It was already known (see for example [4] and references therein) that this quantity is non-trivial and it has a shape quite similar to the one predicted in the mean-field model.

3D SPIN GLASSES: NUMERICAL EVIDENCE FOR A MEAN FIELD LIKE BEHAVIOR AT LOW TEMPERATURE[a]

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We establish evidence that spontaneous replica symmetry breaking theory and not the droplet model describes with good accuracy the equilibrium behavior of the 3d Ising spin glass. We analyze numerical simulations of the static and the dynamical behavior of the system.

1 Introduction

Mean field spin models with quenched disorder are at this point well understood[1,2]. The solution of mean field is non trivial, and new ideas play a very relevant role. The mean field approach predicts the existence of a low temperature glassy phase, characterized by the existence of many different equilibrium states (spontaneous replica symmetry breaking, SRSB). Many new features emerge: states have an ultrametric structure, a small magnetic field does not destroy the spin-glass phase and some observable quantities can have sample-to-sample fluctuations. On the other hand it is possible to define a different consistent theory [3] by starting from the Migdal-Kadanoff approach. By using a common terminology we will refer in the following to this approach as to the droplet model. Here one expects the equilibrium state to be unique (apart from global inversions in zero magnetic field) and that the most relevant excitations are obtained by reversing large domains of spins (the droplets).

There are two different starting points. One is the infinite range approximation which leads to the replica symmetry breaking picture and the other is the Migdal-Kadanoff approximation which leads to the droplet model. Although each of the two pictures is correct in its range of validity we have to establish which of the two qualitatively describes the physics of the real three dimensional spin glasses.

The main result of this work (which continues the investigation started in [4], and follows a long series of Monte Carlo simulations of spin glass systems [5]) has been to gather new and strong evidence that in three dimensions the SRSB picture (and not the droplet model) describes correctly what is observed in numerical simulations.

Let us start by summarizing the evidence we will present in this note and the scheme of our reasoning (for a more detailed exposition of these and more data see

[a] Talk presented by E.M.