Numerical Evidence of a Critical Line in the 4d Ising Spin Glass

Numerical Evidence of a Critical Line in the $4d$ Ising Spin Glass.

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Abstract. – We study numerically the four-dimensional $\pm J$ Ising spin-glass at nonzero external magnetic field. We find numerical evidence of the existence of the Almeida-Thouless critical line. The critical exponents differ from those found at zero external magnetic field.

Nowadays there is great interest in understanding the physical properties of short-range spin glasses. There are two main approaches in order to understand what the nature of the phase transition is and how to describe the spin-glass phase. The first one is the replica approach [1] successfully applied for the SK model [2] and the second one is the phenomenological droplet approach [3]. Their predictions are substantially different. One of the most striking differences between both approaches concerns what the nature of the spin glass phase is. If the replica approach admits the possibility of a spin glass phase with the same features as in the SK model (existence of a lot of thermodynamic phases hierarchically organized and differing in $O(1)$ total free energy) the droplet theory takes as a basic assumption the existence of a unique thermodynamic state.

Quite recently [4], the hypothesis of a unique thermodynamic state has been tested in the $4d$ Ising spin glass with a continuous distribution of Gaussian couplings. Using Monte Carlo numerical simulation it was found that the probability $P(0)$ to have overlap $q = 0$ between two different replicas is nearly constant with the size L of the lattice. This cannot be explained within the droplet approach. Another prediction of the droplet approach concerns

the transition line in the field-temperature (h, T)-plane (the so-called Almeida-Thouless (AT) line in mean-field theory [5]). It has been argued that the spin glass phase cannot survive under application of an external magnetic field.

During the past, the numerical research has been mainly concentrated in the determination of a phase transition at zero external magnetic field. Even though there is no doubt on the existence of a phase transition in the four-dimensional Ising spin glass [6], the situation is less clear in the three-dimensional case [7-9]. Very recently some results have been obtained on the existence of the AT line in $3d$ Ising spin glass [10,11]. The stability of the ordered phase when applying a magnetic field has been investigated and the results suggest that there is no AT line.

In this letter we address the question about the existence of the AT line in the $4d \pm J$ Ising spin glass using Monte Carlo numerical simulation. We have found evidence of a phase transition far from the critical point in the (h, T)-plane which suggests that there is a irreversibility line along which the spin glass susceptibility diverges. The model is defined by

$$H = - \sum_{(i,j)} J_{ij}\,\sigma_i\,\sigma_j - h \sum_i \,\sigma_i \tag{1}$$

and the J_{ij} couple nearest neighbours in a four-dimensional lattice with periodic boundary conditions. The J_{ij} are quenched variables which take the discrete values ± 1 with equal probability. The spins σ_i also take the discrete values ± 1. Monte Carlo simulations make use of the heat bath algorithm. In order to enlarge statistics, 8 identical systems run in parallel each one with the same realization for the disorder.

The overlap among two different replicas with the same realization of the disorder has been computed at each time t:

$$q(t) = \frac{1}{N} \sum_{i=1}^{N} \sigma_i^{(1)}(t)\,\sigma_i^{(2)}(t)\,, \tag{2}$$

where $\{\sigma_i^j;\ j = 1, 2\}$ denotes spins σ_i belonging to replica j. Once the system has been thermalized, one calculates the probability distribution of overlaps $P(q)$ from statistics obtained during an interval of t_0 Monte Carlo steps:

$$P(q) = \overline{\frac{1}{t_0} \sum_{t=1}^{t_0} \delta\,(q - q(t))}\,, \tag{3}$$

where $\overline{(...)}$ means average over the quenched disorder. From this distribution its different moments can be obtained:

$$\overline{\langle q^k \rangle} = \int_{-1}^{1} q^k\,P(q)\,\mathrm{d}q\,. \tag{4}$$

We define the spin glass susceptibility

$$\chi_{\mathrm{SG}} = N(\overline{\langle q^2 \rangle} - \overline{\langle q \rangle}^2)\,. \tag{5}$$

All the information being in the function $P(q)$, the transition temperature and the critical exponents can be found using finite-size scaling as shown in [6]. Near the critical temperature the spin glass susceptibility follows the scaling law

$$\chi_{\mathrm{SG}} = L^{2-\eta}\,\overline{\chi}(L^{1/\nu}(T - T_c))\,. \tag{6}$$

At zero external magnetic field the phase transition can be found using the scaling function g,

$$g = \frac{1}{2}\left(3 - \frac{\overline{\langle q^4 \rangle}}{\overline{\langle q^2 \rangle^2}}\right), \tag{7}$$

which scales like

$$g = \bar{g}(L^{1/\nu}(T - T_c)). \tag{8}$$

We have studied the system at zero external magnetic field for the sizes $L = 3, 4, 5$ and a number of samples 2500, 2500, 1200, respectively. In each case a set of 10 different temperatures have been simulated ranging from $T = 1.5$ up to $T = 2.8$. At zero field we have not used any special cooling procedure, since the samples seemed to thermalize relatively easily near the transition temperature. In general, 15000 Monte Carlo steps where enough to thermalize the system in the range of temperatures and sizes studied.

In fig. 1 we show the g-function equation (7) plotted *vs.* the temperature. From the scaling equation (8) we obtain $T_c = 2.06 \pm 0.02$ and $\nu = 0.7 \pm 0.2$. In fig. 2 we determine the exponent η using the scaling behaviour of the spin glass susceptibility given in eq. (6). We obtain $\eta = -0.25 \pm 0.01$ and from the scaling relation $\gamma = (2 - \eta)\nu$ we obtain $\gamma = 1.6 \pm 0.4$. Our results are in good agreement with those obtained from high-temperature expansions [12] which give $T_c = 2.02 \pm 0.06$ and $\gamma = 2.0 \pm 0.4$. The exponents are also similar (but not equal) to those found in the $4d$ Ising spin glass with Gaussian couplings.

We now present the results for finite applied magnetic field. Using the renormalization group expansion near six dimensions there is no reason why the criterium of universality for the critical exponents should apply in short-range spin glasses [13]. Then, if there is a divergence in the spin glass susceptibility, there are not strong arguments that the critical exponents will be the same as those at zero external field. We have simulated eq. (1) with $h = 0.6$. This is a relatively strong magnetic field. Within mean-field theory and for an applied magnetic field h one would expect to find a phase transition at a temperature given by $\simeq T_{AT}(h/\sqrt{c})$, where c is the connectivity and $T_{AT}(h)$ is the AT line in the SK model. In our case, the transition should be approximately at $T \simeq 1.4$. With such an applied magnetic field
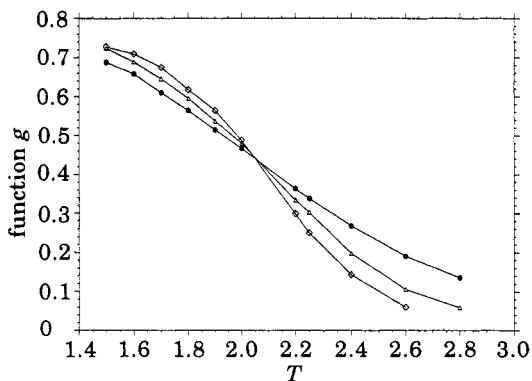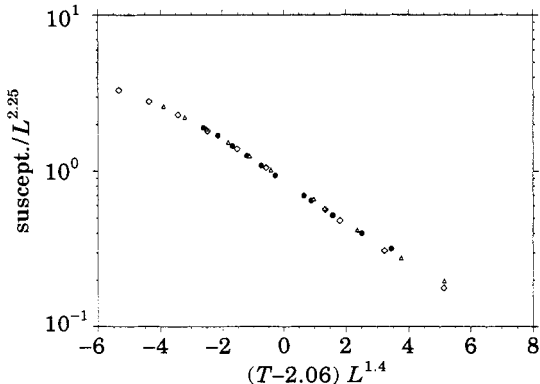

Fig. 1.　　　　　　　　　　　　　　　Fig. 2.

Fig. 1. – Function g at zero magnetic field as defined in eq. (7) plotted *vs.* temperature for three sizes $L = 3, 4, 5$. The symbols are: ● $L = 3$, △ $L = 4$, ◇ $L = 5$.

Fig. 2. – Finite-size scaling for the spin glass susceptibility at zero magnetic field according to eq. (6). The same symbols as in fig. 1.
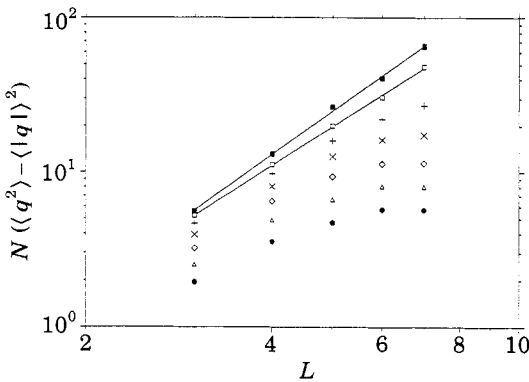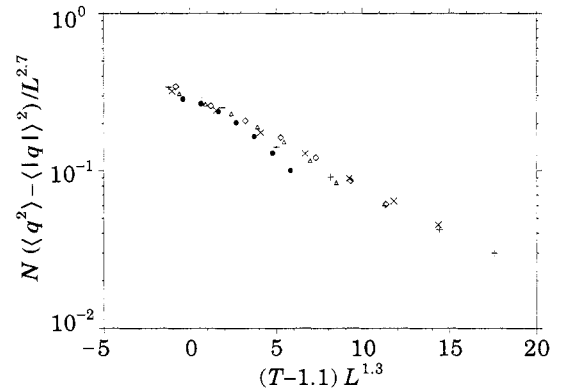
Fig. 3.   Fig. 4.

Fig. 3. – Log-log plot of the spin glass susceptibility in magnetic field $h = 0.6$ defined in eq. (9) vs. size for 8 different temperatures. The symbols are: $\bullet$ $T = 2.5$, $\triangle$ $T = 2.25$, $\diamond$ $T = 2.0$, $\times$ $T = 1.75$, $+$ $T = 1.5$, $\square$ $T = 1.25$, $\blacksquare$ $T = 1.0$.

Fig. 4. – Finite-size scaling for the spin glass susceptibility in magnetic field $h = 0.6$ defined in eq. (9) for different sizes and temperatures. The symbols are: $\bullet$ $L = 3$, $\triangle$ $L = 4$, $\diamond$ $L = 5$, $\times$ $L = 6$, $+$ $L = 7$.

the thermalization becomes very painful because the magnetic field slows the flipping of the spins. In order to be sure we have thermalized the system, we have used a simulated annealing scheme [14]. We considered that we reached full thermalization when the order parameters $\langle q^2 \rangle$ and $\langle q^4 \rangle$ did not experience variation to further increases of Monte Carlo steps in all the steps during the cooling procedure. Five sizes $L = 3, 4, 5, 6, 7$ have been studied. The number of samples ranges from 192 for $L = 3, 4, 5, 6$ up to 64 in case $L = 7$. For all sizes the samples are thermalized at $T = 2.5$ and the temperature is progressively decreased down to $T = 1.0$ in 12 temperature steps of size $\delta T = 0.25$. The different numbers of Monte Carlo steps which the samples stay at each temperature in the annealing process are the following: for $L = 3, 4$ the system stays 10 000 Monte Carlo steps at $T = 2.5$ up to 75 000 at $T = 1.0$, for $L = 4, 5$ the system stays 10 000 Monte Carlo steps at $T = 2.5$ up to 120 000 at $T = 1.0$ and for $L = 7$ the system evolves over 20 000 Monte Carlo steps at $T = 2.5$ up to 120 000 at $T = 1.0$. Furthermore, at each temperature statistics is collected over 10 000 Monte Carlo steps. For the largest size $L = 7$, the system has evolved over approximately half a million Monte Carlo steps until reaching $T = 1.0$.

In order to obtain the spin glass susceptibility equation (5), we have calculated $P(q)$ from eq. (3). Since we are far from T_c and we have a finite size, there is an important contribution to the tail of $P(q)$ corresponding to negative values of the overlap. This effect of the tail would violate the scaling relation equation (6) obtained under the assumption of a unique thermodynamic solution. To suppress this undesirable finite-size effect (which disappears only for very large sizes) we have calculated

$$\chi_{SG} = N(\overline{\langle q^2 \rangle} - \overline{\langle |q| \rangle}^2). \tag{9}$$

Figure 3 shows the spin-glass susceptibility equation (9) at different temperatures and sizes. For $T = 1.1$, χ_{SG} seems to diverge approximately like $L^{2.7}$. This gives $\eta \sim -0.7 (\pm 0.2)$ which is different from the exponent η found at zero magnetic field. Figure 4 shows the scaling relation equation (6) for the spin-glass susceptibility equation (9). Data for $L = 3$ show

a slight deviation from the scaling behaviour possibly because the size is too small or because we have reached temperatures too far from the scaling region (*i.e.* the largest one, $T = 2.5$, is more than two times the estimated critical temperature). A divergence appears at $T \simeq 1.1(\pm 0.2)$ and $\nu \sim 0.8(\pm 0.2)$ and the critical exponents seem to differ from those previously found at $h = 0$. The magnitude of this temperature is close to that found within mean-field theory. In this case (and also in the previous one with zero magnetic field), errors are being estimated by looking at the range of exponents and critical temperatures for which the scaling equation (6) is reasonably satisfied. These procedures give a qualitative estimate.

In summary, we have looked for the AT line in the $4d$ Ising spin glass. At zero external field we find the phase transition and we then obtain critical exponents which are similar to those obtained in the case of Gaussian couplings [6] and are in agreement with those obtained making high-temperature expansions [12]. At $h = 0.6$ we are far from the critical point at zero field and we have found numerical evidence of a divergence of the spin-glass susceptibility defined in eq. (9). In this case, thermalization is very slow but we have been able to equilibrate samples using a simulated annealing procedure. Our results suggest the existence of a phase transition at $T \simeq 1.1$ near the critical temperature predicted within mean-field theory. Our estimates for the critical exponents are different to those found at zero magnetic field. The existence of this transition is an important result, since it contradicts one of the most important consequences of the droplet approach. After completing this work, we have known about a different numerical approach to this problem which supports our main conclusion on the existence of the AT line [15].

The simulations described here were performed in a Transputer machine of 64 nodes (T805) with a peak performance of 100 M flops.

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