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SUPER-ROUGHENING IN TYPE II SUPERCONDUCTORS AND IN GROWTH OF SURFACES IN DISORDERED MEDIA*

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Abstract.
We will study analytical and numerically the two dimensional random phase sine Gordon model. The importance of this model relies on that it describes the growth of surfaces on disordered substrates and flux lines in Type II superconductors in presence of impurities. In particular we will focus the discussion on low temperature properties, showing that the low temperature phase is super-rough.

1. Introduction. The study of the influence of the disorder in materials is a very interesting subject because the samples used in the experiments are not completely pure and also because the properties of the system can change radically.

Theoretically we can use the Harris criteria [1] to study if the disorder changes or not the universality class of the model (i.e. the properties are completely different from the original pure model). In some cases the model is marginal with respect to the Harris criteria (for instance, the Ising model in two and four dimensions with quenched dilution) and we must do numerical simulations or try to solve the model (using, for example, the renormalization group) in order to understand if the universality properties of the model change or not.

The flux lines appear in the Type II superconductors in the so-called mixed phase. Until a strength of the magnetic field (H_1) the Meissner effect holds (i.e. the superconductivity holds) but for $H > H_1$ the magnetic field penetrates in the bulk of the superconductor in small regions (the flux lines). Finally if $H > H_2$ the magnetic field penetrates completely in the bulk of the superconductor destroying the superconductivity properties. Inside the region $H_1 < H < H_2$ the superconductor properties in principle hold, but unfortunately if one applies a potential to the superconductor the electrons that are confined in the flux lines (and that are not bound in Cooper pairs) move and dissipate, and consequently the material lacks its superconductor properties [6].

If we produce defects (for instance oxygen vacancies in the material), these impurities pin the flux lines and so the material recover its superconductor properties [6]. The technological importance is very high because the Type II superconductors present higher values of H_2 than the Type I superconductors (for instance B_c in Type I is roughly some Gauss whereas that B_2 in Type II superconductor is of the order of some Teslas).

The sine Gordon model with random phases describes this physical situation [3] (see also the review [7]), in particular, flux lines in a superconducting film subject to random pinning and parallel magnetic field [3]. Moreover it can describe crystalline surface with disordered substrate [2]. The model has been studied, both numerical and theoretically [2, 4, 3, 5, 16, 1, 9, 13, 14, 17, 19, 18, 20] and in neither area is there agreement on the details of the low temperature phase.

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In this paper we will shown the results that we have obtained in the last years in order to solve the previous controversy. We refer to the interesting reader to references [10, 11, 12] where more detailed studies can be found.

2. Definitions and Analytical results. In this section we fix the notation and write down the different analytical predictions for the observables.

The Hamiltonian for the Laplacian growth of surfaces on disordered substrate is

$$\mathcal{H}[\phi] \equiv \frac{\kappa}{2} \sum_{<ij>} (\phi_i - \phi_j)^2, \qquad \phi_i \equiv d_i + \eta_i \ , \tag{1}$$

where d_i are integral numbers. For the sine Gordon model

$$\mathcal{H}[\phi] \equiv \frac{\kappa}{2} \sum_{<ij>} (\phi_i - \phi_j)^2 - \lambda \sum_i \cos\left(2\pi(\phi_i - \eta_i)\right) \ . \tag{2}$$

In both cases η is a quenched noise uniformly distributed in the interval $[-\frac{1}{2}, \frac{1}{2}]$.

We can relate both models by means of the Poisson summation formula. Another easy method is to take the limit $\lambda \to \infty$ in the continuum model (sine Gordon model).

We can compute the probability distribution to have a given difference of the fields

$$\mathcal{P}[\Delta(r), T] \equiv \overline{\langle \delta\left[\Delta(\mathbf{r}) - (\phi(\mathbf{r_0}) - \phi(\mathbf{r_0} + \mathbf{r}))\right]\rangle} \ . \tag{3}$$

Where $\langle(\cdots)\rangle$ denotes thermodynamical average at fixed disorder and $\overline{(\cdots)}$ denotes the average over the quenched disorder.

The second moment of (3) is the correlation function (or propagator) of the fields (or heights in the terminology of the surfaces)

$$C(r, T) \equiv \overline{\langle(\phi(\mathbf{r_0}) - \phi(\mathbf{r_0} + \mathbf{r}))^2\rangle} \ , \tag{4}$$

and the fourth cumulant (or four point function) is

$$D(r, T) \equiv 3\left(\overline{\langle\Delta^2(r)\rangle}\right)^2 - \overline{\langle\Delta^4(r)\rangle} \ . \tag{5}$$

At this point we can compute the Binder cumulant

$$B(r, T) \equiv \frac{1}{2}\left(3 - \frac{\overline{\langle\Delta^4(r)\rangle}}{(\overline{\langle\Delta^2(r)\rangle})^2}\right) \ . \tag{6}$$

If the probability distribution is Gaussian (i.e. the underlying effective Hamiltonian is Gaussian) then $C(r)$ will grow logarithmically and both D and B should be zero.

In the next paragraphs we will describe the analytical prediction for the observables defined above using the Renormalization Group approach (RG) and a variational computation.

The starting point for the analytical studies is the average of the replicated Hamiltonian in the limit of zero replicas (the replica trick, see [1] for more details). The model in the high-T region is Gaussian. The sine-term (< 1) does not affect to the kinetic term and so the theory is completely free. The propagator is (for $T > T_c$)

$$C_{T>T_c}(r, T) \simeq \frac{T}{\kappa\pi} \log r \ . \tag{7}$$

The discrepancies arise in the study of the low temperature properties of the model. For instance the RG predicts (for $T < T_c$)[2, 1, 16, 4, 3]

$$C^{\rm RG}_{T<T_c}(r,T) \simeq a \log r + b \log^2 r \ , \qquad b = \frac{2}{\pi^2}\left(\frac{T_c - T}{T_c}\right)^2 \ , \tag{8}$$

whereas a variational computation show [9, 13, 14] [1]

$$C^{\rm VAR}_{T<T_c}(r,T) \simeq \frac{T_c}{\kappa\pi} \log r \ , \tag{12}$$

In the both cases the critical temperature is the same: $T_c = \kappa/\pi$. It is easy to show, for instance using the Harris criteria, that the critical temperature is independent of the value of λ.

In the framework of the RG it is possible to compute higher moments of the probability distribution. For instance the fourth momenta reads [12]

$$\overline{\langle(\Phi(r) - \Phi(0))^4\rangle} = 3\left(\frac{4\rho_*}{\kappa}\log r + \frac{\beta_{*g}}{\kappa^2}(\log r)^2\right)^2 - 48 d_* \log r \ , \tag{13}$$

where ρ_*, β_{*g} and d_* are constants. The function D is given by [12]

$$D^{\rm RG}_{T<T_c}(r,T) = 48 d_* \log r \ , \tag{14}$$

and finally the Binder cumulant reads [12]

$$B^{\rm RG}_{T<T_c}(r,T) = \frac{24 d_* \log r}{(\frac{4\rho_*}{\kappa}\log r + \frac{\beta_{g*}}{\kappa^2}(\log r)^2)^2} \simeq_{r\gg 1} \frac{24 d_* \kappa^2}{\beta_g^* \log^3 r} \ . \tag{15}$$

This last equation says us that the probability distribution of the correlation functions computed at large distances is asymptotically Gaussian, but this is different from the fact that at any finite distance the probability distribution is Gaussian.

We can write the following Langevin equation in order to study the dynamic of the model (with no conserved parameter)[5]

$$\frac{\partial\phi(x)}{\partial t} = -\frac{\delta\beta\mathcal{H}}{\delta\phi(x)} + \rho(x,t), \tag{16}$$

[1] In the replicated action we can write the most general Gaussian action (neglecting in the notation the replica indexes) (see for instance [8])

$$S_{\rm G} = \frac{1}{2}\int {\rm d}^2 p \ \phi(p) G(p) \phi(-p) \ . \tag{9}$$

The variational parameter is the propagator itself of the theory. The free energy in this approximation is ($F_{\rm VAR}$)

$$F \leq F_{\rm VAR} \equiv F_G + \langle S - S_G\rangle_G, \tag{10}$$

where F_G is the free energy of the action S_G and $\langle(\cdots\rangle_G$ denotes average with the Gaussian action S_G. F is the free energy of the original model. The variational condition on the propagator is

$$\frac{\delta}{\delta G(p)}\left[F_G + \langle S - S_G\rangle_G\right] = 0 \ . \tag{11}$$

where ρ is a Gaussian noise, with variance proportional to the temperature and zero mean. Using the Martin-Siggia-Rose formalism and the RG it is possible to obtain the behavior of the dynamical critical exponent with the temperature [5]

$$z(T) = 2 + 2e^{\gamma}(1 - T/T_c), \tag{17}$$

for $T < T_c$ and near of the transition point. γ is the Euler constant and $T_c = \kappa/\pi$. In the rough phase the model is described by a Gaussian theory and z should take the Gaussian vale ($z = 2$).

3. Numerical Results. In this section we will discuss the results that we have obtained. Firstly we will show the results of the static of the system and finally we will study the dynamic (for more details see references [10, 11, 12]). All the numerical results have been obtained using the supercomputer APE-100 [26].

We have focused our static numerical studies on the study of the propagator $C(x)$. In the analysis of this object we have used the exact analytical expression for the propagator for a Gaussian theory, $P_L(r)$, defined in a box:

$$P_L(r) = \frac{1}{2L^2} \sum_{n_1=1}^{L-1} \sum_{n_2=0}^{L-1} \frac{1 - \cos(\frac{2\pi r n_1}{L})}{2 - \cos(\frac{2\pi n_1}{L}) - \cos(\frac{2\pi n_2}{L})} \simeq \frac{1}{2\pi} \log(\frac{r}{2\sqrt{2}e^{\gamma}}) \ , \tag{18}$$

where the symbol $\simeq$ holds for $L \gg 1$ and $r \gg 1$. Only for large distance it is well approximate by the logarithm. If one try to analyze the numerical data using the logarithm must fight against two factors: i) the propagator becomes the logarithm only for large distances, and ii) the propagator does not diverge indefinitely because the system is finite with periodic boundary conditions: $C(x) = C(L - x)$. In some analysis of the numerical data done in the literature [19] forget the previous items and consequently the conclusions are wrong [2].

In order to examine the behavior of the propagator in the low temperature phase we try to fit the numerical propagator to

$$C(r) = b_1 P_L(r) + b_2 P_L(r)^2 \tag{19}$$

where we have used that the representation of the lattice of $\log^2(r)$ is given by $P_L(r)^2$, and so the information of the possible super-roughening of the low-T phase is encoded in the b_2 coefficient. In this representation the prediction of the renormalization group is $b_2 = \frac{8}{\pi^2}\tau^2$ where $\tau \equiv (T - T_c)/T_c$ is the reduced temperature.

We have plotted in figure 1 the results of the fit for three different values of λ and in a wide range of temperatures. It is clear that b_2 is zero in the rough phase (Gaussian phase) and then becomes to be non zero qualitatively following the RG predictions. For this plot it is evident that if we define as the "apparent" critical temperature the temperature where b_2 is non zero, this "apparent" critical temperature depends on the value of the coupling against of the Renormalization Group predictions. In the conclusion we will examine again this problem. Initially the agreement between our numerical simulations and the RG predictions was only qualitative. In particular the value of the coefficient of τ^2 that we have found was a factor 5 less than the RG prediction. But lastly, Carpentier and Le Doussal claim [15] (and compute the exact number on a triangular lattice) that the original RG prediction is wrong in a factor

[2] Another example: if one correlation function goes like $\exp(-r/\xi)$ in a finite lattice with periodic boundary conditions the fit should be $\cosh((x - L/2)/\xi)$ due the periodic boundary conditions.

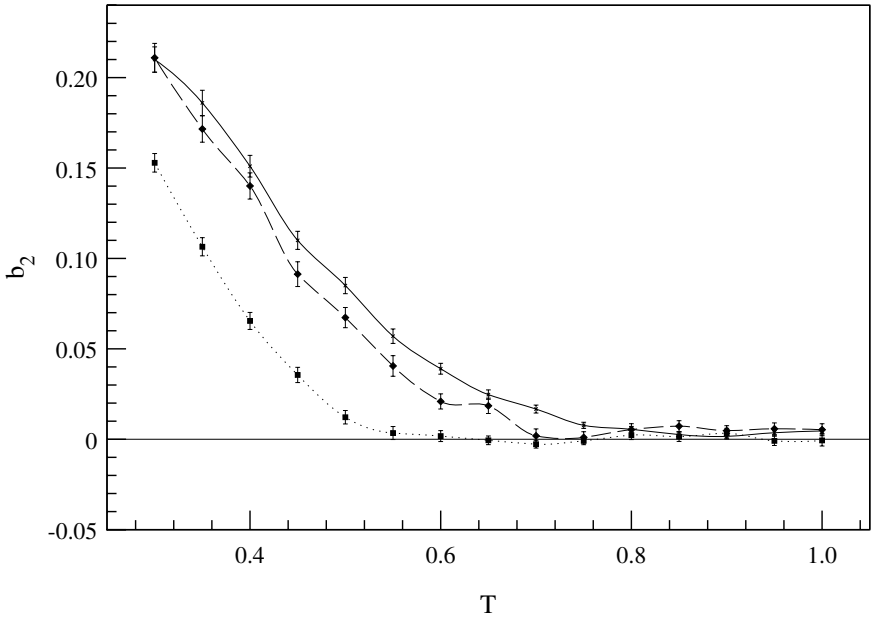

FIG. 1. *Coefficient of the* $\log^2$ *(b_2) versus the temperature for different values of the λ parameter from a two parameter fit. Left to right in the plot: $\lambda = 0.5, 2.0$ and $\lambda = \infty$ (discrete model).*

4 (minor algebraic mistakes in the older RG calculations)[15], and so the agreement turns out to be quantitative.

The b_1 exponent initially follows the predictions of the Gaussian model (that it is the effective theory of the high-T, phase-rough): i.e. it is proportional to the temperature, and near of the transition left the linear dependence of the temperature.

Another interesting test of the RG prediction is to study the D (see (14). We recall again that the RG prediction, for $T < T_c$, is a logarithmic growth (i.e. proportional to $P_L(r)$) in the super-rough phase and zero in the rough phase while the Variational methods predicts that both, D and B, should be zero (at least for large distances).

In figure 2 we show the $D(r)$ function in the rough phase (lower curve) that for $r < 7$ is zero and we can see that in the super-rough phase (upper curve) grows proportionally to the Gaussian propagator, according with the RG. In particular this last $D(r)$ curve can be fitted as $10^4 D = (114 \pm 3)P(r) + (36 \pm 3)$.

Similar results have been obtained calculating the ground state of the discrete model. It is possible to show that the computation of the ground state of the discrete model is equivalent to solve a problem of combinatorial optimization [21, 22]. By calculating the propagator using those ground states it is possible to see that the propagator at $T = 0$ is super-rough [23, 24]. The values of the b_2 coefficient differs from those of the extrapolation of the RG predictions up to $T = 0$, but there are no reason to claim the validity of the RG far away of the critical region. Similar results at $T = 0$ was obtained by finding the ground state solving the Langevin equation at $T = 0$ (i.e. a deterministic equation) [25].

In the rest of this section we will study the dynamical properties of the sine Gordon model with random phases.

It is possible, knowing the equilibrium propagator at a given temperature, to compute a dynamical critical correlation length ($\xi(t)$) that can be thought as the

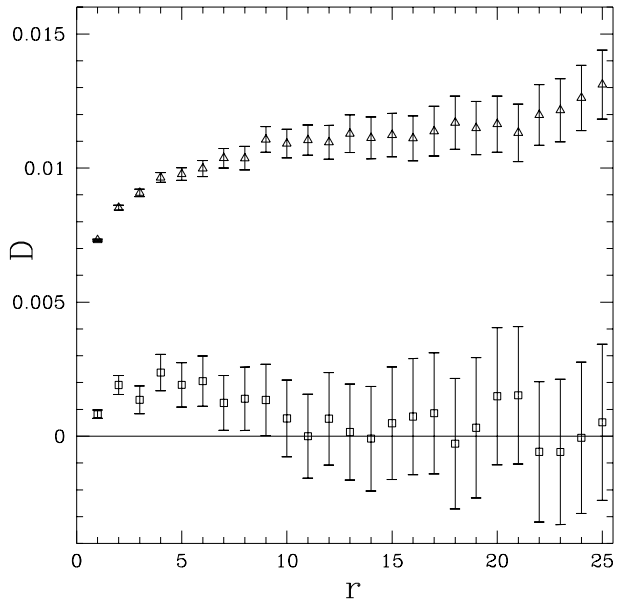

FIG. 2. *D as a function of r at $T = 0.40$ ($\triangle$) and $T = 1.0$ ($\square$) for $L = 64$. here $\lambda = \infty$ (discrete model)*

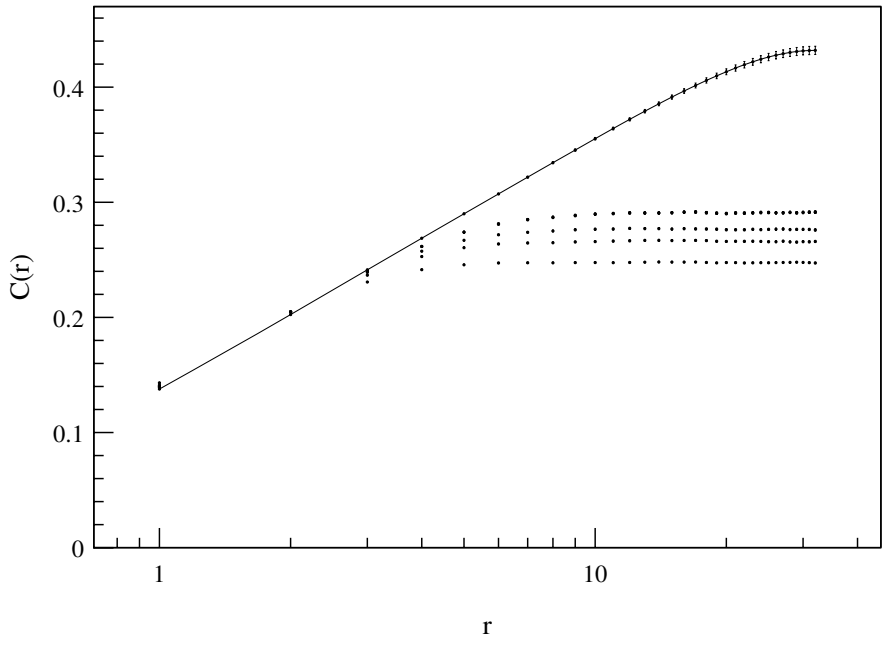

FIG. 3. *Correlation functions, $C_t(r)$, shown at equal time intervals (bottom to top: t =2000, 4000, 6000 and 8000 sweeps), and the equilibrium correlator, $C_{\rm asy}(r)$, as the continuous line ($\lambda =$ 2.0, $T = 0.5$).*

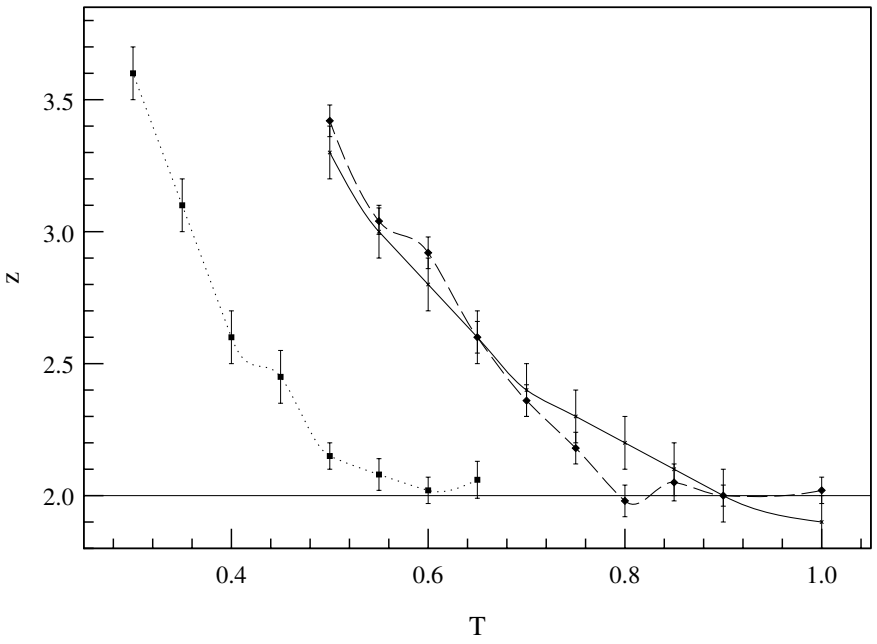

FIG. 4. *The dynamical critical exponent, z, against temperature for $\lambda = 0.5, 2.0$ and ∞. The Gaussian value is marked with a horizontal line. We use squares and a dotted line for $\lambda = 0.5$, diamonds and a dashed line for $\lambda = 2.0$ and crosses and a continuous line for $\lambda = \infty$. The lines are only to guide the eye.*

distance up to which the system is thermalized at time t. The structure of the propagator in the off equilibrium regime (i.e. it reaches a plateau before arriving at the half of the lattice) permits us to define this dynamical correlation length as follow. If we denote as $C_{\rm asyn}(r)$ the equilibrium correlation length at a given temperature and $C_t(r)$ is the propagator measured at the time t, the dynamical correlation length is defined as $C_{\rm asyn}(\xi(t)) = \text{Plateau}[C_t(r)]$. In the figure 3 we show the equilibrium correlation function (top line) and different correlation functions measured at different times (still in the off equilibrium regime). Roughly $\xi(t)$ is the point where the equilibrium propagator and the propagator measured at time t begin to differ.

We have checked that with this definition the dynamical correlation length follows a power law with the time. The exponent of this power law is just the inverse of the dynamical critical exponent (z):

$$\xi(t) \propto t^{1/z} \ . \tag{20}$$

Obviously where the system is Gaussian we should recover the Gaussian value for z: $z = 2$. In the super-rough phase, asumming RG, we should see a linear dependence on the reduced temperature (with slope 3.56).

We have shown our data on the dependence on the temperature of the z exponent in figure 4. Again, if we define an "apparent" critical temperature, in which the z exponent begins to be different from 2, it is clear the dependence on λ of this "apparent" critical temperature. This effect is similar to that of b_2. The data begin to differ from the Gaussian value following a behavior that can be fitted assuming a linear dependence. The result is that the slope is more o less independent of the λ value and in agreement with the RG value: 3.56.

4. Conclusions. From the previous section it is clear that the low temperature phase of the random phase sine Gordon model is super-rough according with the predictions of the (replica symmetric) renormalization group even to the quantitative level.

Moreover the dynamical critical transition takes place just at the static one and again is well described by the renormalization group..

Open problems to study is that the "apparent" critical temperature seems to depend on the coupling, but this problem could be explained in the framework of the renormalization group, in particular showing that for the values of λ simulated the running coupling constant at scale L are still far away of the fixed point. Moreover, a theoretical finite size study of the RG equation would be welcome.

Another interesting open problem is the study of the super-rough phase to check that this phase is really described by a replica symmetric theory: i.e. the super-rough phase is not glassy.

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