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The confining–Higgs phase transition in U(1)-Higgs LGT

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We simulate the compact U(1)-Higgs model in a four-dimensional lattice. We present a numerical study for the behaviour around the region where the transition between the confining and Higgs phases disappears. The transition line is found to be first order and to end in a second-order point. We measure the critical exponents of the endpoint obtaining the mean field ones within errors.

The study of the critical behaviour of lattice gauge theories in four dimensions is crucial to formulate nonperturbatively the underlying continuum quantum field theory. Even if a continuum limit is possible, the question remains whether the obtained theory is trivial or not.

For the selfcoupled scalar field with a quartic term in the action (the $\lambda\phi^4$ theory) no other critical point in $d=4$ has been found than an infrared stable gaussian fixed point at the origin $\lambda=0$, $m=0$. This means that the only fixed point with infinite correlation length, where continuum theories may be defined, is also where the theory is noninteracting (trivial).

The critical behaviour of this model around the fixed point is perturbatively accessible, because at that point the coupling vanishes. The situation changes when the critical point corresponds to nonvanishing bare couplings, as it is the case we study in this letter, and therefore a nonperturbative method, like Monte Carlo, is mandatory.

When gauge degrees of freedom are added, as in the SU(2)×U(1)-Higgs model, the phase transition structure of the parameter space becomes very rich. In particular, it is very important to know the critical properties of the second order points as they may lead to bounds on parameters such as the Higgs mass. A full discussion can be found in ref. [1].

In this letter, we present some results of a numerical study of the compact U(1) gauge field coupled to a charged scalar field with fixed modulus ($|\Phi|^2=1$).

We use the standard action

$$S=-\beta\sum_{\text{plaqs.}}\operatorname{Re}U_{\text{plaqs.}}-\kappa\sum_{\text{links}}\operatorname{Re}\Phi_r^{\dagger}U_{r\to r'}\Phi_{r'}\,. \qquad (1)$$

where both the gauge and the scalar field are U(1) variables.

The two-parameter space (β, κ) corresponding to the action (1) is known to present at most three different phases (see fig. 1) [2,3]. The point A in fig. 1 represents a first-order transition [4] and the point B ($\beta=\infty$) a second-order one [5]. The line A–C is first order while the line B–C is, for a scalar Higgs field of modulus one, second order. We are interested in the study of the transition region between the Higgs and confining phases, which in our case are analytically connected. More precisely, we want to know the nature of the transition line D–C and especially of the endpoint D.

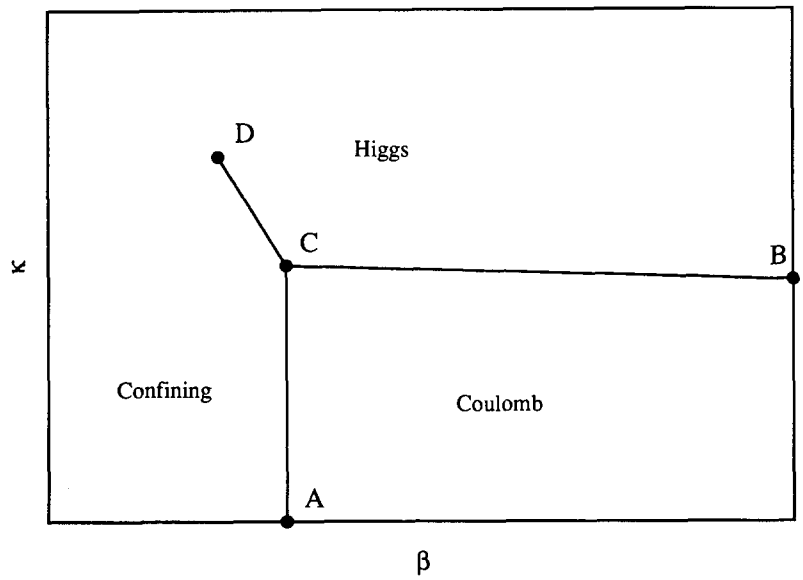

Fig. 1. Schematic representation of the parameter space in U(1)-Higgs lattice gauge theory.

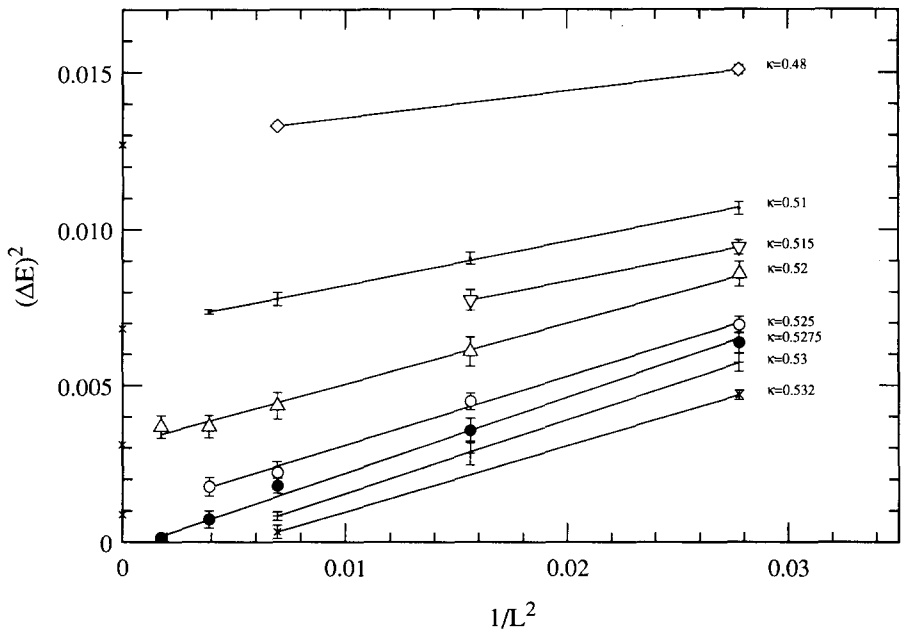

Fig. 2. Latent heat ΔE (squared) as a function of $1/\sqrt{V}$. We find an almost linear behaviour that allows us to compute with accuracy the latent heat in the thermodynamic limit. It is nonzero – first order transition – for $\kappa < \kappa_c \approx 0.5275$.

The attainment of numerical values for the critical exponents at D compatible with the classical ones will give a strong indication for a non-interacting theory in the continuum.

We simulate a system with the action (1) in lattices of sizes ranging from 6^4 to 24^4. We use a standard adaptive Metropolis algorithm with discretized U(1) variables belonging to the Z_{1024} group. We measure both the plaquette and link energies and store them individually to construct the corresponding

histograms. Typically we perform 100 000–500 000 iterations at each point, with up to 1 000 000 in some points.

For each value of the κ coupling we use the spectral density method [6] to compute the probability distribution for values of β close to the one we used in the simulation. In this way we are able to locate as precisely as possible the point on the transition line, without having to explore a two-dimensional region of the parameter space.

For each simulated point we compute the plaquette energy histogram. We find distributions with one or two peaks. In the latter case we can adjust the heights of the maxima using the spectral density method and, then, measure the distance between the peaks. This quantity may be seen as the latent heat of the phase transition. A nonvanishing value in the thermodynamic limit is a clear signature of its first-order character.

The spectral density method can also be used to move in the two-dimensional parameter space. It is possible to combine both the plaquette and link energies to evaluate the fluctuation matrix from which we can compute its eigenvalues and eigenvectors. A detailed analysis of the whole measured data is in preparation and will be presented elsewhere [7]. In this letter, we will limit ourselves to the discussion of the results obtained from the latent heat of the plaquette energy as well as the dispersion in that energy.

First let us consider the thermodynamic limit for a given value of κ. In fig. 2 we plot the square of the latent heat as a function of the inverse of the square root of the lattice volume. The statistical errors have been computed by a jack-knife method. We observe a clear nonzero $V\to\infty$ limit for $\kappa\leqslant 0.525$, so we can conclude that the transition line D–C is first order near D. We have also performed some simulations far from D in the same line checking that the first order nature remains along the whole line. For $\kappa\geqslant 0.5275$ we find that the jump vanishes before the $V\to\infty$ limit.

The pattern that we obtain – a first-order transition ending at a finite value of the parameters – is similar to that of the 2D Ising model with nonvanishing magnetic field or of the liquid–vapour system. In our system the plaquette energy behaves as an order parameter, playing the role of the magnetization in the Ising model, with a jump at the transition line equal to the latent heat. The dispersion of the plaquette energy in the absence of phase coexistence may then be related with the dispersion of the magnetization, or magnetic susceptibility in the Ising case. Notice however, that we could have chosen other order parameters like the link energy or a linear combination of both energies, giving equivalent results regarding the critical behaviour. We refer to ref. [7] for a detailed discussion on this subject.

If we use κ to parametrize the C–D line, we conclude that the evolution of the latent heat ΔE will follow the power law

$$\Delta E = A(\kappa_c - \kappa)^{\beta}, \tag{2}$$

with β the usual magnetization exponent.

For every lattice size, we fit the obtained values of ΔE to the function (2). We stress the dependence of κ_c on L (see figs. 3 and 4). We have used κ in the interval [0.51, 0.532]. Our result for the exponent β from the $L=16$ data is

$$\beta = 0.47(9)\,. \tag{3}$$

To reduce the statistical errors we have fitted simultaneously the $L=12$ and 16 data with a single β value obtaining

$$\beta = 0.54(6)\,. \tag{4}$$

The values (3), (4) point towards the classical value $\beta=0.5$. In fig. 3 (upper side) we plot the square of the latent heat as a function of κ, which clearly shows a linear behaviour.

In fig. 3 (lower side) we plot the "susceptibility" χ, i.e. the dispersion in the plaquette energy. We can check the second-order nature of the point D from the divergence in χ. We have fitted our $L\to\infty$ estimations to the law $\chi\propto|\kappa-\kappa_c|^{-\gamma}$ finding the value 1.04 but with a large error (about 20%). The solid line in the lower part of fig. 3 is a fit to a linear divergence.

The exponent ν can also be computed directly from the shift in the apparent critical point κ_c^L as a function of L. We compute from a fit to the function (2) with the same β for all lattices. Taking a fixed value $\beta=\frac{1}{2}$ we obtain

$$\nu = 0.47(4)\,. \tag{5}$$

Changing β in the interval [0.4, 0.6] we observe vari-

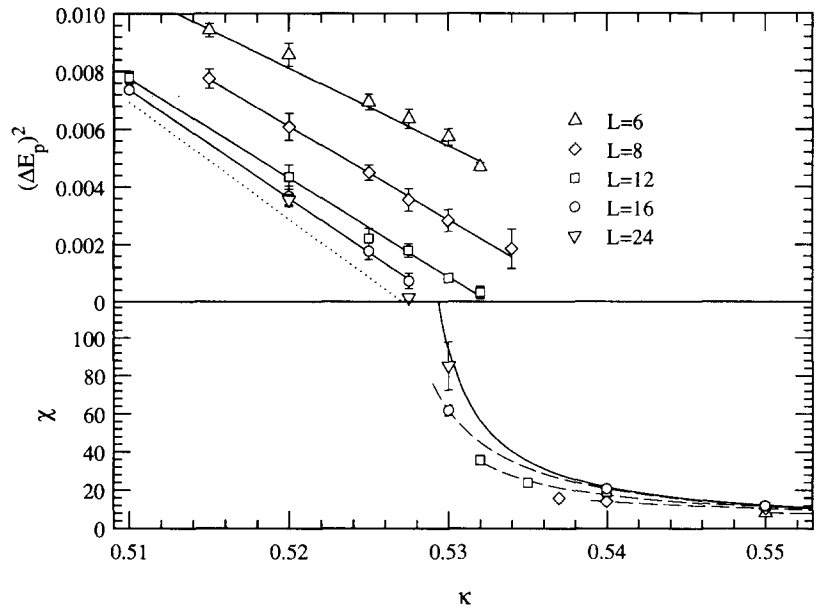

Fig. 3. Latent heat squared as a function of κ (upper side). The dotted line is the infinite volume limit. On the lower side we plot the plaquette energy susceptibility as a function of κ, when $\kappa > \kappa_c^L$ (absence of phase coexistence).

ations in the value of ν smaller than the statistical error reported in (5).

In fig. 4 we plot κ_c^L as a function of $1/L^2$. The linear fit (dashed line) corresponds to the value $\nu = \frac{1}{2}$.

The location of the endpoint D and the angle of the first-order line (with the β axis) are found to be

$$\kappa_c^\infty = 0.5267(6)\,, \tag{6}$$

$$\beta_c^\infty = 0.8484(10)\,, \tag{7}$$

$$\theta = -0.8045(10)\,. \tag{8}$$

We remark that the variation of the angle of the

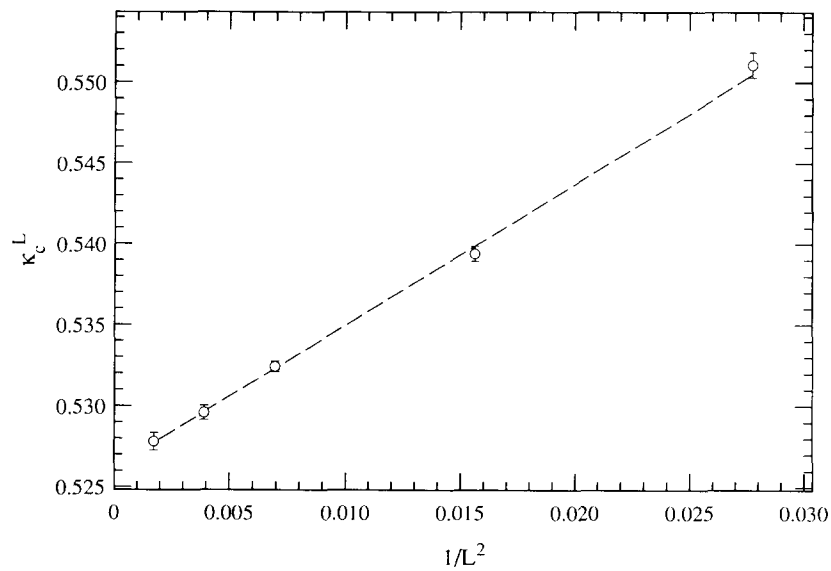

Fig. 4. Apparent critical point κ_c^L as a function of $1/\sqrt{V}$. The straight line corresponds to the value $\nu = \frac{1}{2}$.

first-order transition line in the region $\kappa \in [0.51, 0.5275]$ is less than 1%.

As a result of the previous analysis we conclude that at the end point D there is a second-order phase transition with critical exponents compatible with the mean field ones within errors.

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