The Coulomb-Higgs phase transition in Z_8 and $q = 8$ U(1)-Higgs models

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The Coulomb-Higgs phase transition of the four dimensional Z_8 gauge model is studied. We find clear first order properties in contradiction with the previously stressed second order behavior. That transition point may be regarded as the end of a transition line of the U(1)-Higgs model with charge $q = 8$, that has been also assumed of second order. We show that this line is first order in a wide region, with a decreasing latent heat that goes to zero when closing to the $4\!-\!d$ XY model limit.

1. Introduction

Abelian gauge models have been the first systems studied in four dimensions by means of Monte Carlo Simulations [1].

The discrete Z_N subgroups of U(1), in addition of being convenient approximations of the continuous group, present very interesting features in spite of their simplicity.

The Z_2 gauge model has a first order transition [2], between two massive regions: the small β regime (strong coupling) where electric charge is confined, known as the Confining phase, and the large β region where magnetic charge is confined and electric charge is free [3], known as the Higgs phase.

On the other hand, the U(1) gauge model, that can be considered as the $N \to \infty$ limit of Z_N, has two phases: an electric confining phase and a massless Coulomb phase where electric and magnetic charges are free. The last one being qualitatively different from the Z_2 Higgs phase.

The study of intermediate Z_N models has supplied the connection between these two different behaviors. In fact, for $N \geq 5$ the four dimensional Z_N gauge model undergoes two phase transitions [3, 4]. The first transition, occurring at small (close to 1) β, is almost N-independent and separates a disorder Confining phase, with an area behavior for the decay of the Wilson Loop (electric charges confined), from a disorder Coulomb phase with a perimeter decay of the Wilson Loop and zero mass (free photons). The second transition separates the Coulomb phase from a Higgs phase where electric charge is free, magnetic monopoles are confined, and the mass gap is different from zero. For $N = 2, 3, 4$ there is only a first order phase transition [5].

From a more technical point of view, Z_N groups have been extensively used in order to improve the speed of Monte Carlo simulations of U(1) on the lattice [6], provided the two Z_N transitions are clearly separated.

More recently these models have been studied because of their relationship with the Gauge-Higgs systems. In fact, if we consider the U(1) gauge model interacting with Higgs fields of charge N, in the limit of large gauge-Higgs coupling κ, this model becomes Z_N. For finite κ the transition continues in the $\beta - \kappa$ plane [7–9], ending (for $\beta \to \infty$) in the critical point of the XY model. This point is a very well established second order phase transition [10].

The duality transformation plays a central role in the computations of the Z_N models. In fact these models (with the Wilson action) are self-dual [11] for $N = 2, 3, 4$ while for $N \geq 5$ the self-duality is only approximate [3] [4].

The order of the phase transitions of Z_N with $N \geq 5$ has been a controversial subject. The small β transition at large N was initially considered of second or higher order [6], but further analysis has shown its first order nature [12]. The Coulomb-Higgs transition has also been considered up to now of second order, but the quasi self-duality of the model points out that both transitions should be of the same order.

We will study in this letter the Coulomb-Higgs transition, in the $N = 8$ case, by computing the specific heat and the shift in the apparent critical coupling, obtaining results which are compatible with a first order picture of the transition. We have also been able to compute directly the latent heat within a 5% precision.

We also consider the U(1)-Higgs model with charge $q = 8$, and study the line, previously assumed of second order, emerging from the Z_8 transition. We will find that our results are compatible with first order at least in a sensible region of the parameter space.

2. The Z_8 model

We use the standard Wilson action

$$S = -\beta \sum_{\mathbf{r},\mu<\nu} \mathrm{Re}\, U_{\mathbf{r},\mu\nu}, \qquad (1)$$

where $\mathbf{r}$ is the four dimensional lattice site, and the indices μ, ν represent the four lattice directions. $U_{\mathbf{r},\mu\nu}$ is the plaquette defined by the site $\mathbf{r}$ and the directions μ and ν.

The Z_N gauge model in four dimensions for $N \geq 5$ presents two phase transitions: one at $\beta \approx 1$, related with the deconfinement phenomena, and the other one at [6] $\beta \approx 0.78/(1 - \cos\frac{2\pi}{N})$ that appears as an effect of the discrete nature of the group.

We choose the $N = 8$ model as representative of the set of groups for $N \geq 5$. Notice that increasing the value of N moves the transition to the region of large β where the system is more difficult to thermalize.

We use a standard 3-hit Metropolis algorithm. We have simulated on lattice sizes ranging from $L = 8$ to $L = 24$. We have run on WorkStations for the smaller lattices and on the RTN machine [13] for the $L = 16$ and $L = 24$ ones. The total number of thousands of Monte Carlo iterations have been 800, 170, 260 and 75 for $L = 8$, 12, 16, and 24, respectively, and some thousands more for thermalization and location of the critical point.

We start our simulations from cold configurations. We measure the energy of the system as well as the Polyakov loop. We do not use the latter for a determination of the relevant thermodynamic quantities, however its evolution is useful in order to find the critical region. For $\beta < \beta_c$ the phase of the Polyakov loop changes rapidly in Monte Carlo time, while for $\beta > \beta_c$ it is almost constant.