

Numerical matrix method for quantum periodic potentials

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Mathematica version...

```
In[1]:= StringJoin[{ "The running Mathematica version is ", ToString[$Version]}]
```

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Out[1]= The running Mathematica version is
10.1.0 for Microsoft Windows (64-bit) (March 24, 2015)
```

Surface states

Functions that for a KP potential with lattice parameter $a=1$ and n_b barriers:

- (I) evaluate the Hamiltonian matrix H_{nm} [Eq. (17)]
- (II) solve Eq. (10) to find the eigenenergies and eigenfunctions.

The size of the system is $L=n_b*a=n_b$.

The positions of the nb barriers are stored in the vector (or list) xrV , and the corresponding widths and heights are stored in vectors bV and $V0V$, respectively.

Note that these vectors xrV , bV and $V0V$ describe the periodic potential.

We use units such as $\hbar^2/2\mu=1$, μ being the mass of the particle.

Functions F_{nm} [Eq. (20)]

```
In[2]:= f[k_, x_, L_] := f[k, x, L] = Sin[k * Pi * x / L] / (Pi * k)
```

```
In[3]:= Fnm[n_, m_, x_, L_] := f[m - n, x, L] - f[m + n, x, L]
```

```
In[4]:= Fnn[n_, x_, L_] := x / L - f[2 * n, x, L]
```

Functions h_{nm} [Eq. (18)]

```
In[5]:= hnm[n_, m_, s_, b_, L_] := Fnm[n, m, s + b / 2, L] - Fnm[n, m, s - b / 2, L]
```

```
In[6]:= hnn[n_, s_, b_, L_] := Fnn[n, s + b / 2, L] - Fnn[n, s - b / 2, L]
```

Matrix elements H_{nm} [Eq. (17)]

```
In[7]:= HHnn[n_, L_, V0V_, xrV_, bV_] := N[(n * Pi / L)^2 +
  Sum[V0V[[k]] * hnn[n, xrV[[k]], bV[[k]], L], {k, 1, Length[xrV]}]]
```

```
In[8]:= HHnm[n_, m_, L_, V0V_, xrV_, bV_] :=
  Sum[V0V[[k]] * hnm[n, m, xrV[[k]], bV[[k]], L], {k, 1, Length[xrV]}] // N
```

The function *HamiltonianMatrix* provides the $N \times N$ Hamiltonian matrix for the KP potential given by xrV , bV and $V0V$, where $N=Nterms$

```
In[9]:= HamiltonianMatrix[Nterms_, L_, V0V_, xrV_, bV_] :=
  Table[If[n == m, HHnn[n, L, V0V, xrV, bV], HHnm[n, m, L, V0V, xrV, bV]],
  {n, 1, Nterms}, {m, 1, Nterms}]
```

The function *soluKP* provides the solution of Eq. (13).

The list of energies is provided directly by the function *EnV*. The n-th element of this matrix is the eigenvalue E_n

The matrix of coefficients $c_m^{(n)}$ of Eq. (14) are provided by *cnMatrix*. The element (n,m) of this matrix is just the coefficient $c_m^{(n)}$.

```
In[10]:= soluKP[Nterms_, L_, V0V_, xrV_, bV_] := soluKP[Nterms, L, V0V, xrV, bV] =
  Eigensystem[N[HamiltonianMatrix[Nterms, L, V0V, xrV, bV]]] // Chop //
  Transpose // Sort // Transpose
```

```
In[11]:= EnV[Nterms_, L_, V0V_, xrV_, bV_] :=
  EnV[Nterms, L, V0V, xrV, bV] = soluKP[Nterms, L, V0V, xrV, bV][[1]]
```

```
In[12]:= cnMatrix[Nterms_, L_, V0V_, xrV_, bV_] :=
  cnMatrix[Nterms, L, V0V, xrV, bV] = soluKP[Nterms, L, V0V, xrV, bV][[2]]
```

The function *psi* provides an estimate of n-th eigenfunction according the formula

$$\psi_n(x) \simeq \sum_{m=1}^{N^*} c_m^{(n)} \varphi_m(x)$$

When $Nast=N^*$ is equal to $Nterm=N$, this expression is just Eq. (14).

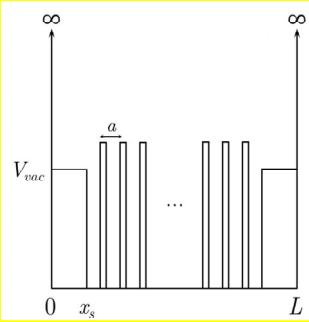
The function *ProbDensity* is just $|\psi_n(x)|^2$

```
In[13]:= psi[n_, x_, Nast_, Nterms_, L_, V0V_, xrV_, bV_] := Sqrt[2 / L] *
  Sum[cnMatrix[Nterms, L, V0V, xrV, bV][[n, m]] * Sin[m * Pi * x / L], {m, 1, Nast}]
```

```
In[14]:= ProbDensity[n_, x_, Nast_, Nterms_, L_, V0V_, xrV_, bV_] :=
  Abs[psi[n, x, Nast, Nterms, L, V0V, xrV, bV]]^2
```

Definition of the dimerized KP system and plot of the potential for the case with $b=1/6$ considered in Fig. 8

We generate the list of positions, widths and heights of the barriers that define the KP potential of Fig. 7:



In this *example* the internal $nb=10$ barriers of width $b=q=1/6$ and height $V_0=v0=2P/q=120$ [$P=10$] are placed at $x=2,3,\dots,11$.

Besides we stick two barriers of height $V_{vac}=Vvac=50$ and width $1+q/2$ to the walls of the infinite square well of size $L=nb+3=13$

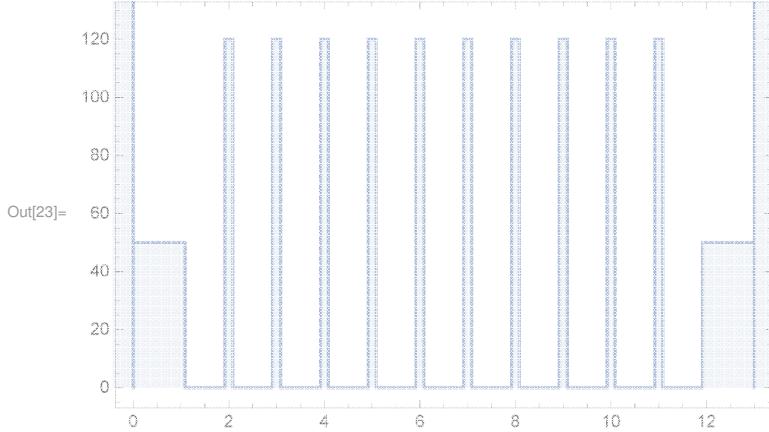
```
In[15]:= Vvac = 50;
nb = 10; L = N[nb] + 3; PP = 10;
q = 1./6; v0 = 2.*PP/q;
{xrVector = Table[k, {k, 1, Round[L - 1]}], bVector = Table[q, {k, 1, Round[L - 1]}],
 V0Vector = Table[v0, {k, 1, Round[L - 1]}]};
xrVector[[1]] = (1 + q/2.) / 2.; bVector[[1]] = 1 + q/2.;
xrVector[[Length[xrVector]]] = L - (1 + q/2) / 2;
bVector[[Length[xrVector]]] = 1 + q/2.;
V0Vector[[1]] = Vvac; V0Vector[[Length[xrVector]]] = Vvac;
```

The next instruction generates a plot of the potential defined above

```

leftWall = Plot[If[x < 0, 1.2 * v0, 0],
  {x, -1, 0.01}, Filling -> Bottom, PlotPoints -> 100, Frame -> True];
rightWall = Plot[If[x > L, 1.2 * v0, 0], {x, L - 0.01, L + 1},
  Filling -> Bottom, PlotPoints -> 100, Frame -> True];
KPPotential = Plot[Piecewise[Table[{V0Vector[[k]],
  xrVector[[k]] - bVector[[k]] / 2 <= x <= xrVector[[k]] + bVector[[k]] / 2},
  {k, 1, L - 1}]], {x, 0, L}, Filling -> Bottom, PlotPoints -> 4000, Frame -> True];
Show[leftWall, rightWall, KPPotential, PlotRange -> {{-0.1, L + 0.1}, {0, 1.05 v0}}]

```



Evaluation of the functions $R(x) = |\Psi_1(x)|^2 / |\Psi_1(0)|^2$ for the cases considered in Fig. 8: $n_b=10$, $V_{\text{vac}}=50$, $P=10$, $N=400$ and $V_0=2P/b$, with $b=1/6$, $1/12$ and $1/96$

Case with $b=1/6$

We define the system

```

In[24]:= q = 1. / 6;
PP = 10; v0 = 2. * PP / q;
Vvac = 50;
nb = 10; L = N[nb] + 3;
{xrVector = Table[k, {k, 1, Round[L - 1]}], bVector = Table[q, {k, 1, Round[L - 1]}],
 V0Vector = Table[v0, {k, 1, Round[L - 1]}]};
xrVector[[1]] = (1 + q / 2.) / 2.; bVector[[1]] = 1 + q / 2.;
xrVector[[Length[xrVector]]] = L - (1 + q / 2) / 2;
bVector[[Length[xrVector]]] = 1 + q / 2.;
V0Vector[[1]] = Vvac; V0Vector[[Length[xrVector]]] = Vvac;

```

The vector of the first Nterms energies E_n is calculated by means of the function EnV

```
In[32]:= Nterms = 400; Energies = EnV[Nterms, L, v0Vector, xrVector, bVector]

Out[32]= {8.22397, 8.22959, 8.40981, 8.59389, 8.84349, 9.14209, 9.47005, 9.80279, 10.1111,
10.3632, 10.5293, 31.9122, 31.9178, 33.0978, 33.826, 34.8436, 36.0656, 37.4144,
38.8026, 40.1226, 41.2404, 42.0031, 56.8054, 56.8054, 68.7958, 68.7989,
72.8295, 74.374, 76.554, 79.1377, 81.937, 84.7461, 87.2903, 89.3355, 91.055,
92.7735, 94.2494, 116.149, 116.154, 126.676, 129.352, 132.912, 136.837,
140.801, 144.692, 148.662, 152.935, 157.525, 162.155, 166.152, 181.916,
182.012, 195.452, 200.115, 205.626, 211.372, 217.151, 222.993, 229.051,
235.454, 242.216, 249.24, 256.261, 267.687, 268.943, 282.015, 289.339,
296.98, 304.677, 312.402, 320.214, 328.201, 336.43, 344.927, 353.684, 362.71,
372.752, 378.327, 389.664, 399.093, 408.515, 417.978, 427.512, 437.142,
446.885, 456.753, 466.744, 476.852, 487.055, 497.25, 508.72, 518.9, 529.62,
540.534, 551.605, 562.807, 574.112, 585.489, 596.907, 608.343, 619.777,
631.167, 642.203, 658.447, 668.514, 680.414, 692.774, 705.39, 718.178,
731.073, 744.019, 756.969, 769.893, 782.773, 795.57, 807.949, 826.918,
837.801, 851.097, 864.954, 879.1, 893.428, 907.864, 922.35, 936.843, 951.318,
965.77, 980.172, 994.252, 1014.1, 1026.63, 1041.54, 1056.94, 1072.6, 1088.44,
1104.38, 1120.39, 1136.43, 1152.5, 1168.6, 1184.72, 1200.69, 1220.27, 1235.16,
1251.78, 1268.73, 1285.88, 1303.19, 1320.61, 1338.12, 1355.72, 1373.39,
1391.15, 1408.99, 1426.88, 1445.84, 1463.53, 1481.84, 1500.33, 1518.95, 1537.7,
1556.58, 1575.59, 1594.73, 1613.99, 1633.38, 1652.91, 1672.62, 1691.24,
1711.79, 1731.73, 1751.72, 1771.81, 1792.01, 1812.35, 1832.85, 1853.5,
1874.33, 1895.31, 1916.45, 1937.86, 1956.78, 1979.87, 2001.36, 2022.86,
2044.42, 2066.1, 2087.91, 2109.9, 2132.07, 2154.42, 2176.95, 2199.65, 2222.62,
2242.61, 2267.65, 2290.67, 2313.68, 2336.76, 2359.93, 2383.25, 2406.74,
2430.42, 2454.28, 2478.3, 2502.49, 2526.93, 2548.75, 2575.09, 2599.63, 2624.17,
2648.78, 2673.5, 2698.35, 2723.35, 2748.53, 2773.86, 2799.36, 2825., 2850.84,
2875.07, 2902.2, 2928.24, 2954.33, 2980.5, 3006.78, 3033.18, 3059.71, 3086.38,
3113.19, 3140.13, 3167.2, 3194.41, 3221.35, 3249.03, 3276.56, 3304.19,
3331.93, 3359.78, 3387.74, 3415.8, 3443.97, 3472.25, 3500.64, 3529.14,
3557.72, 3587.36, 3615.62, 3644.63, 3673.8, 3703.1, 3732.52, 3762.03, 3791.63,
3821.31, 3851.08, 3880.93, 3910.88, 3940.88, 3972.87, 4001.99, 4032.48,
4063.18, 4094.03, 4124.99, 4156.05, 4187.18, 4218.39, 4249.66, 4281.01,
4312.45, 4343.94, 4377.76, 4408.13, 4440.12, 4472.34, 4504.72, 4537.21,
4569.8, 4602.46, 4635.2, 4668., 4700.89, 4733.86, 4766.89, 4801.98, 4834.04,
4867.56, 4901.29, 4935.17, 4969.17, 5003.27, 5037.46, 5071.73, 5106.08,
5140.53, 5175.09, 5209.71, 5245.63, 5279.73, 5314.79, 5350.02, 5385.39,
5420.88, 5456.48, 5492.19, 5527.99, 5563.91, 5599.93, 5636.06, 5672.29,
5708.85, 5745.18, 5781.78, 5818.51, 5855.36, 5892.33, 5929.42, 5966.64,
6003.98, 6041.45, 6079.04, 6116.78, 6154.63, 6191.84, 6230.43, 6268.56,
6306.78, 6345.11, 6383.56, 6422.15, 6460.87, 6499.74, 6538.75, 6577.9,
6617.17, 6656.59, 6694.74, 6735.36, 6775.02, 6814.75, 6854.57, 6894.52,
6934.6, 6974.83, 7015.22, 7055.76, 7096.44, 7137.32, 7178.28, 7217.73,
7260.09, 7301.28, 7342.51, 7383.85, 7425.3, 7466.9, 7508.64, 7550.53, 7592.58,
7634.76, 7677.07, 7719.53, 7760.7, 7804.41, 7847.12, 7889.89, 7932.75,
7975.74, 8018.85, 8062.11, 8105.51, 8149.06, 8192.73, 8236.74, 8280.67,
```

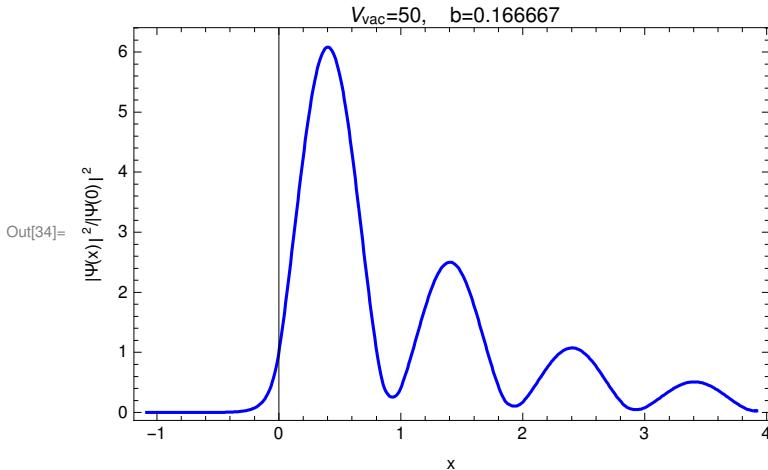
```
8323.88, 8368.67, 8412.9, 8457.21, 8501.63, 8546.16, 8590.81, 8635.58,
8680.48, 8725.51, 8770.66, 8815.93, 8861.32, 8906.75, 8952.44, 8998.17,
9044.04, 9090.01, 9136.12, 9182.31, 9228.67, 9275.08, 9321.74, 9368.37}
```

RxVector is a table of {x,R(x)} with x=0.01, 0.02,...,5

```
In[33]:= xsLeft = 1 + q / 2;
fp1 = ProbDensity[1, xsLeft, Nterms, Nterms, L, V0Vector, xrVector, bVector];
RxVector = Table[{x - xsLeft, ProbDensity[1, x, Nterms,
Nterms, L, V0Vector, xrVector, bVector] / fp1}, {x, 0, 5, 0.01}];
```

RxPlot[Vvac,q] is the plot of R(x) for the case with $V_{\text{vac}} = V_{\text{vac}}$ and $b=q$ (in this case $V_{\text{vac}}=50$ and $q=1/6$)

```
In[34]:= RxPlot[Vvac, q] = ListPlot[RxVector, Frame → True,
FrameLabel → {"x", "|\Psi(x)|^2/|\Psi(0)|^2"}, PlotRange → All,
PlotLabel → StringJoin[{" Vvac=", ToString[Vvac], ", ", "b=", ToString[q]}],
PlotStyle → Blue, Joined → True]
```



Case with $b=1/12$

We define the system

```
In[35]:= q = 1. / 12;
PP = 10; v0 = 2. * PP / q;
Vvac = 50;
nb = 10; L = N[nb] + 3;
xrVector = Table[k, {k, 1, Round[L - 1]}], bVector = Table[q, {k, 1, Round[L - 1]}],
V0Vector = Table[v0, {k, 1, Round[L - 1]}];
xrVector[[1]] = (1 + q / 2.) / 2.; bVector[[1]] = 1 + q / 2.;
xrVector[[Length[xrVector]]] = L - (1 + q / 2) / 2;
bVector[[Length[bVector]]] = 1 + q / 2.;
V0Vector[[1]] = Vvac; V0Vector[[Length[xrVector]]] = Vvac;
```

The vector of the first Nterms energies E_n is calculated by means of the function EnV

```
In[43]:= Nterms = 400; Energies = EnV[Nterms, L, v0Vector, xrVector, bVector]
Out[43]= {7.37018, 7.37226, 7.6405, 7.83377, 8.10474, 8.43315, 8.79748, 9.17083, 9.52038,
9.80912, 10.0008, 28.9946, 28.9966, 30.6839, 31.4203, 32.4739, 33.7475,
35.1555, 36.6071, 37.9914, 39.1689, 39.9766, 55.9362, 55.9362, 65.9341,
65.9383, 69.5575, 71.1196, 73.3149, 75.9188, 78.7613, 81.6808, 84.4849,
86.9075, 88.6339, 89.639, 90.2856, 114.067, 114.07, 124.802, 127.322, 130.748,
134.578, 138.406, 141.957, 145.327, 148.908, 152.817, 156.676, 159.741,
180.83, 180.844, 197.224, 200.99, 205.793, 210.881, 215.884, 220.85, 226.113,
231.866, 237.964, 243.997, 249.121, 267.347, 267.434, 287.559, 292.92, 299.333,
305.945, 312.52, 319.237, 326.395, 334.092, 342.18, 350.342, 357.851, 373.902,
374.3, 396.474, 403.742, 411.934, 420.238, 428.543, 437.063, 446.045, 455.558,
465.497, 475.648, 485.563, 500.423, 501.707, 524.569, 533.935, 543.958,
554.018, 564.102, 574.417, 585.161, 596.397, 608.057, 620.007, 632.043,
646.715, 649.761, 672.311, 683.8, 695.636, 707.458, 719.32, 731.401, 743.856,
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1672.15, 1692.06, 1711.69, 1731.61, 1751.66, 1771.83, 1792.12, 1812.51,
1832.98, 1853.53, 1874.16, 1894.87, 1915.65, 1936.45, 1959.63, 1979.5,
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2359.84, 2383.37, 2406.92, 2430.48, 2454.03, 2477.61, 2501.23, 2524.75,
2553.27, 2574.53, 2598.79, 2623.46, 2648.34, 2673.35, 2698.42, 2723.51,
2748.57, 2773.63, 2798.68, 2823.78, 2848.78, 2879.31, 2901.69, 2927.46,
2953.66, 2980.08, 3006.62, 3033.23, 3059.83, 3086.42, 3112.99, 3139.56,
3166.16, 3192.68, 3224.81, 3248.59, 3275.89, 3303.61, 3331.55, 3359.62,
3387.75, 3415.88, 3443.99, 3472.09, 3500.2, 3528.37, 3556.46, 3589.82,
3615.26, 3644.11, 3673.35, 3702.8, 3732.38, 3762.01, 3791.66, 3821.31,
3850.95, 3880.62, 3910.35, 3940.05, 3974.35, 4001.67, 4032.07, 4062.83,
4093.77, 4124.85, 4155.99, 4187.16, 4218.34, 4249.54, 4280.79, 4312.13,
4343.45, 4378.49, 4407.87, 4439.83, 4472.09, 4504.53, 4537.08, 4569.72,
4602.4, 4635.12, 4667.89, 4700.72, 4733.63, 4766.59, 4802.23, 4833.81,
4867.32, 4901.08, 4934.99, 4969.03, 5003.15, 5037.35, 5071.61, 5105.95,
5140.36, 5174.92, 5209.51, 5245.69, 5279.56, 5314.62, 5349.87, 5385.25,
5420.76, 5456.37, 5492.08, 5527.88, 5563.78, 5599.78, 5635.89, 5672.11,
5708.84, 5745.02, 5781.62, 5818.35, 5855.21, 5892.19, 5929.28, 5966.5,
6003.84, 6041.31, 6078.89, 6116.66, 6154.49, 6191.83, 6230.32, 6268.44,
6306.66, 6344.99, 6383.43, 6422.02, 6460.75, 6499.62, 6538.64, 6577.79,
6617.07, 6656.49, 6694.59, 6735.28, 6774.92, 6814.63, 6854.43, 6894.36,
6934.43, 6974.67, 7015.07, 7055.65, 7096.36, 7137.32, 7178.31, 7217.28,
```

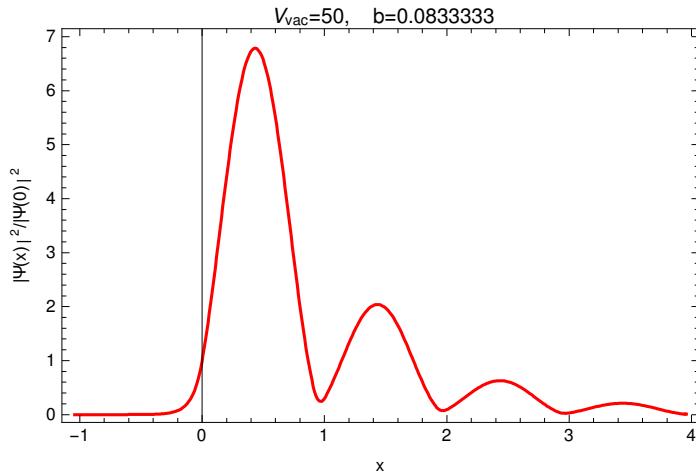
```
7260.1, 7301.26, 7342.45, 7383.73, 7425.15, 7466.72, 7508.46, 7550.39, 7592.5,
7634.76, 7677.16, 7719.71, 7759.77, 7804.51, 7847.18, 7889.88, 7932.66,
7975.56, 8018.63, 8061.89, 8105.34, 8148.99, 8192.79, 8236.97, 8281.06,
8322.33, 8368.92, 8413.11, 8457.29, 8501.58, 8545.99, 8590.57, 8635.34,
8680.32, 8725.48, 8770.81, 8816.27, 8861.88, 8904.61, 8952.77, 8998.47,
9044.21, 9090.01, 9135.98, 9182.07, 9228.42, 9274.91, 9321.73, 9368.56}
```

RxVector is a table of {x,R(x)} with x=0.01, 0.02,...,5

```
In[44]:= xsLeft = 1 + q/2;
fp1 = ProbDensity[1, xsLeft, Nterms, Nterms, L, V0Vector, xrVector, bVector];
RxVector = Table[{x - xsLeft, ProbDensity[1, x, Nterms,
Nterms, L, V0Vector, xrVector, bVector] / fp1}, {x, 0, 5, 0.01}];
```

RxPlot[Vvac,q] is the plot of R(x) for the case with $V_{\text{vac}} = V_{\text{vac}}$ and $b = q$ (in this case $V_{\text{vac}} = 50$ and $q = 1/12$)

```
In[45]:= RxPlot[Vvac, q] = ListPlot[RxVector, Frame → True,
FrameLabel → {"x", "|Ψ(x)|²/|Ψ(0)|²"}, PlotRange → All,
PlotLabel → StringJoin[{" Vvac=", ToString[Vvac], ", b=", ToString[q]}],
PlotStyle → Red, Joined → True]
```



Case with $b = 1/96$

We define the system

```
In[46]:= q = 1./96;
PP = 10; v0 = 2.*PP/q;
Vvac = 50;
nb = 10; L = N[nb] + 3;
{xrVector = Table[k, {k, 1, Round[L - 1]}], bVector = Table[q, {k, 1, Round[L - 1]}],
v0Vector = Table[v0, {k, 1, Round[L - 1]}]};
xrVector[[1]] = (1 + q/2.)/2.; bVector[[1]] = 1 + q/2.;
xrVector[[Length[xrVector]]] = L - (1 + q/2)/2;
bVector[[Length[xrVector]]] = 1 + q/2.;
v0Vector[[1]] = Vvac; v0Vector[[Length[xrVector]]] = Vvac;
```

The vector of the first Nterms energies E_n is calculated by means of the function EnV

```
In[54]:= Nterms = 400; Energies = EnV[Nterms, L, v0Vector, xrVector, bVector]
Out[54]= {6.7741, 6.77502, 7.14114, 7.34667, 7.64246, 8.00563, 8.41293, 8.83503, 9.23517,
9.56957, 9.79401, 26.859, 26.8622, 29.0242, 29.8038, 30.9273, 32.3008, 33.8246,
35.406, 36.9296, 38.2444, 39.1602, 54.5901, 54.5902, 64.0419, 64.0535,
66.7784, 68.4643, 70.7873, 73.5259, 76.5147, 79.6029, 82.6201, 85.3391,
87.4715, 88.8032, 89.0582, 111.581, 111.59, 121.544, 124.235, 127.838,
131.907, 136.032, 139.85, 143.215, 146.498, 150.099, 153.785, 156.769,
178.736, 178.752, 194.334, 198.203, 203.142, 208.379, 213.443, 218.242,
223.15, 228.541, 234.34, 240.064, 244.808, 265.538, 265.595, 285.959, 291.122,
297.382, 303.86, 310.146, 316.412, 323.1, 330.398, 338.101, 345.744, 352.365,
372.19, 372.343, 396.623, 403.213, 410.94, 418.766, 426.455, 434.306, 442.731,
451.814, 461.326, 470.836, 479.425, 498.739, 499.093, 526.804, 534.871,
544.017, 553.264, 562.435, 571.895, 582.01, 592.813, 604.071, 615.397,
625.954, 645.133, 645.824, 676.39, 686.013, 696.707, 707.42, 718.113, 729.179,
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1371.19, 1389.35, 1407.32, 1428.77, 1432.32, 1471.61, 1487.61, 1504.41,
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1674.12, 1678.67, 1719.79, 1737.37, 1755.65, 1773.94, 1792.37, 1811.36,
1831.14, 1851.68, 1872.78, 1894.19, 1915.55, 1939.17, 1944.84, 1987.54,
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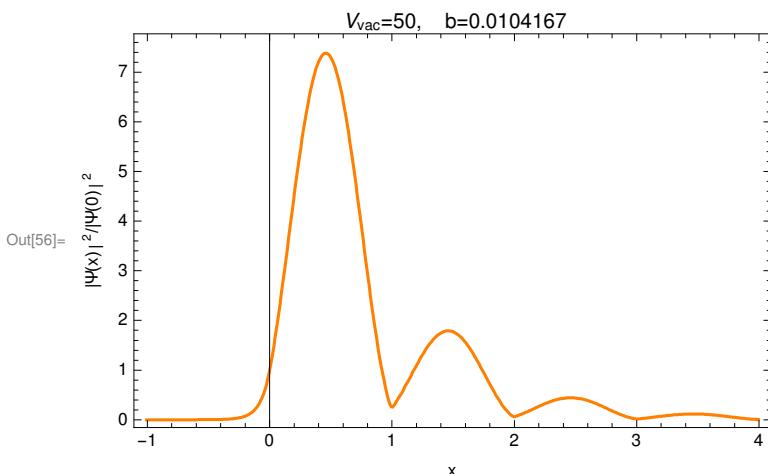
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4770.3, 4786.66, 4841.05, 4872.94, 4905.11, 4937.22, 4969.59, 5002.56,
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6237.18, 6273.73, 6310.36, 6347.08, 6384.07, 6421.66, 6460.03, 6499.17,
6538.9, 6579.01, 6619.23, 6659.58, 6681.86, 6741.83, 6779.97, 6818.27,
6856.53, 6895.06, 6934.18, 6974.09, 7014.75, 7056.02, 7097.67, 7139.58,
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7550.14, 7592.94, 7636.12, 7679.4, 7722.72, 7747.97, 7810.84, 7852.07,
7893.42, 7934.74, 7976.34, 8018.54, 8061.52, 8105.25, 8149.6, 8194.31,
8239.45, 8284.25, 8310.66, 8375.7, 8418.4, 8460.79, 8503.62, 8546.74, 8590.48,
8635., 8680.28, 8726.16, 8772.42, 8818.77, 8865.07, 8893.1, 8959.27, 9003.56,
9047.98, 9092.39, 9137.11, 9182.45, 9228.62, 9275.53, 9323.28, 9371.34}
```

RxVector is a table of {x,R(x)} with x=0.01, 0.02,...,5

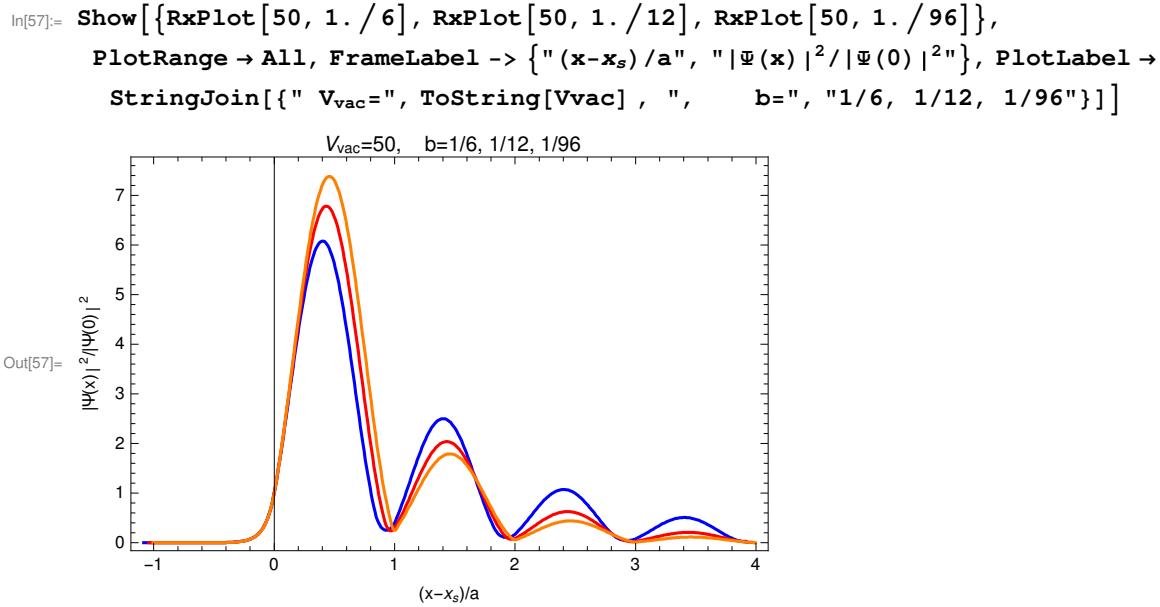
```
In[55]:= xsLeft = 1 + q / 2;
fp1 = ProbDensity[1, xsLeft, Nterms, Nterms, L, V0Vector, xrVector, bVector ];
RxVector = Table[{x - xsLeft, ProbDensity[1, x, Nterms,
Nterms, L, V0Vector, xrVector, bVector ] / fp1}, {x, 0, 5, 0.01}];
```

RxPlot[Vvac,q] is the plot of R(x) for the case with $V_{\text{vac}} = V_{\text{vac}}$ and $b=q$ (in this case $V_{\text{vac}}=50$ and $q=1/96$)

```
In[56]:= RxPlot[Vvac, q] = ListPlot[RxVector, Frame → True,
FrameLabel → {"x", "|Ψ(x)|^2 / |Ψ(0)|^2"}, PlotRange → All,
PlotLabel → StringJoin[{" V_{\text{vac}}=", ToString[Vvac], ", b=", ToString[q]}],
PlotStyle → Orange, Joined → True]
```



We combine and compare the function $R(x) = |\Psi_1(x)|^2 / |\Psi_1(0)|^2$ for these three cases



Relative probability $R(x) = |\Psi_1(x)|^2 / |\Psi_1(0)|^2$ for several values of V_{vac}
when $n_b=10$, $P=10$, $V_0=2P/b$ with $b=1/96$, and $N=400$

We define the system and the value of N

```
In[58]:= Nterms = 400;
q = 1./96;
PP = 10; v0 = 2.*PP/q;
nb = 10; L = N[nb] + 3;
xrVector = Table[k, {k, 1, Round[L - 1]}], bVector = Table[q, {k, 1, Round[L - 1]}],
v0Vector = Table[v0, {k, 1, Round[L - 1]}];
xrVector[[1]] = (1 + q/2.)/2.; bVector[[1]] = 1 + q/2.;
xrVector[[Length[xrVector]]] = L - (1 + q/2.)/2;
bVector[[Length[xrVector]]] = 1 + q/2.;
```

R(x) for $V_{\text{vac}}=50$

```
In[65]:= Vvac = 50;
v0Vector[[1]] = Vvac; v0Vector[[Length[xrVector]]] = Vvac;
```

The vector of the first Nterms energies E_n is calculated by means of the function EnV

```
In[67]:= Energies = EnV[Nterms, L, v0Vector, xrVector, bVector]
Out[67]= {6.7741, 6.77502, 7.14114, 7.34667, 7.64246, 8.00563, 8.41293, 8.83503, 9.23517,
9.56957, 9.79401, 26.859, 26.8622, 29.0242, 29.8038, 30.9273, 32.3008, 33.8246,
35.406, 36.9296, 38.2444, 39.1602, 54.5901, 54.5902, 64.0419, 64.0535,
66.7784, 68.4643, 70.7873, 73.5259, 76.5147, 79.6029, 82.6201, 85.3391,
87.4715, 88.8032, 89.0582, 111.581, 111.59, 121.544, 124.235, 127.838,
```

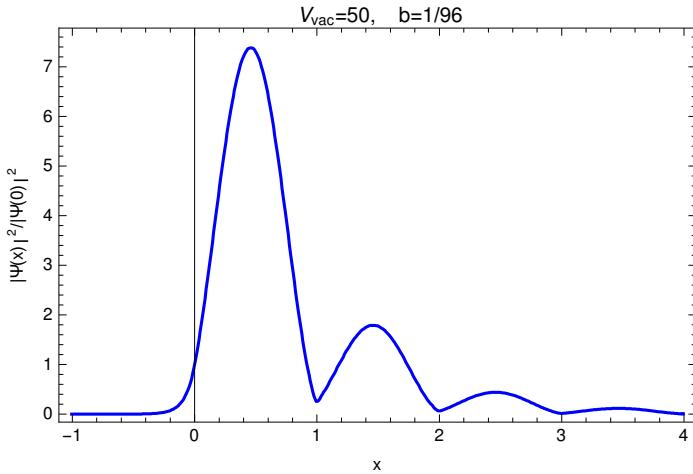
131.907, 136.032, 139.85, 143.215, 146.498, 150.099, 153.785, 156.769, 178.736, 178.752, 194.334, 198.203, 203.142, 208.379, 213.443, 218.242, 223.15, 228.541, 234.34, 240.064, 244.808, 265.538, 265.595, 285.959, 291.122, 297.382, 303.86, 310.146, 316.412, 323.1, 330.398, 338.101, 345.744, 352.365, 372.19, 372.343, 396.623, 403.213, 410.94, 418.766, 426.455, 434.306, 442.731, 451.814, 461.326, 470.836, 479.425, 498.739, 499.093, 526.804, 534.871, 544.017, 553.264, 562.435, 571.895, 582.01, 592.813, 604.071, 615.397, 625.954, 645.133, 645.824, 676.39, 686.013, 696.707, 707.42, 718.113, 729.179, 740.951, 753.429, 766.394, 779.486, 791.998, 811.375, 812.555, 845.734, 856.914, 869.07, 881.275, 893.507, 906.173, 919.575, 933.702, 948.335, 963.152, 977.533, 997.392, 999.226, 1034.61, 1047.39, 1061.13, 1074.84, 1088.62, 1102.88, 1117.9, 1133.65, 1149.93, 1166.43, 1182.65, 1203.22, 1205.83, 1243.33, 1257.7, 1272.91, 1288.14, 1303.47, 1319.31, 1335.93, 1353.28, 1371.19, 1389.35, 1407.32, 1428.77, 1432.32, 1471.61, 1487.61, 1504.41, 1521.17, 1538.05, 1555.47, 1573.67, 1592.62, 1612.13, 1631.93, 1651.64, 1674.12, 1678.67, 1719.79, 1737.37, 1755.65, 1773.94, 1792.37, 1811.36, 1831.14, 1851.68, 1872.78, 1894.19, 1915.55, 1939.17, 1944.84, 1987.54, 2006.74, 2026.62, 2046.44, 2066.43, 2086.99, 2108.33, 2130.44, 2153.13, 2176.14, 2199.18, 2224.01, 2230.84, 2275.2, 2295.99, 2317.33, 2338.69, 2360.23, 2382.35, 2405.26, 2428.94, 2453.2, 2477.8, 2502.43, 2528.53, 2536.63, 2582.44, 2604.84, 2627.78, 2650.68, 2673.77, 2697.44, 2721.91, 2747.15, 2772.98, 2799.16, 2825.43, 2852.85, 2862.21, 2909.59, 2933.56, 2957.98, 2982.41, 3007.05, 3032.28, 3058.31, 3085.1, 3112.49, 3140.24, 3168.05, 3196.85, 3207.56, 3256.31, 3281.89, 3307.91, 3333.88, 3360.07, 3386.85, 3414.43, 3442.77, 3471.72, 3501.03, 3530.47, 3560.65, 3572.7, 3622.97, 3650.12, 3677.6, 3705.11, 3732.84, 3761.17, 3790.3, 3820.19, 3850.69, 3881.55, 3912.5, 3944.1, 3957.59, 4009.18, 4037.93, 4067.02, 4096.06, 4125.34, 4155.22, 4185.89, 4217.33, 4249.38, 4281.79, 4314.36, 4347.38, 4362.25, 4415.36, 4445.65, 4476.2, 4506.78, 4537.6, 4569.03, 4601.24, 4634.22, 4667.81, 4701.77, 4735.83, 4770.3, 4786.66, 4841.05, 4872.94, 4905.11, 4937.22, 4969.59, 5002.56, 5036.32, 5070.84, 5105.96, 5141.46, 5177.15, 5213.07, 5230.83, 5286.75, 5320.18, 5353.78, 5387.43, 5421.34, 5455.85, 5491.15, 5527.21, 5563.87, 5600.91, 5638.06, 5675.46, 5694.75, 5751.93, 5786.95, 5822.18, 5857.37, 5892.82, 5928.87, 5965.71, 6003.3, 6041.5, 6080.08, 6118.87, 6157.73, 6178.44, 6237.18, 6273.73, 6310.36, 6347.08, 6384.07, 6421.66, 6460.03, 6499.17, 6538.9, 6579.01, 6619.23, 6659.58, 6681.86, 6741.83, 6779.97, 6818.27, 6856.53, 6895.06, 6934.18, 6974.09, 7014.75, 7056.02, 7097.67, 7139.58, 7181.4, 7205.05, 7266.72, 7306.36, 7345.98, 7385.77, 7425.83, 7466.5, 7507.94, 7550.14, 7592.94, 7636.12, 7679.4, 7722.72, 7747.97, 7810.84, 7852.07, 7893.42, 7934.74, 7976.34, 8018.54, 8061.52, 8105.25, 8149.6, 8194.31, 8239.45, 8284.25, 8310.66, 8375.7, 8418.4, 8460.79, 8503.62, 8546.74, 8590.48, 8635., 8680.28, 8726.16, 8772.42, 8818.77, 8865.07, 8893.1, 8959.27, 9003.56, 9047.98, 9092.39, 9137.11, 9182.45, 9228.62, 9275.53, 9323.28, 9371.34}

RxVector is a table of {x,R(x)} with x=0.01, 0.02,...,5

```
In[68]:= xsLeft = 1 + q / 2;
fp1 = ProbDensity[1, xsLeft, Nterms, Nterms, L, V0Vector, xrVector, bVector];
RxVector = Table[{x - xsLeft, ProbDensity[1, x, Nterms,
Nterms, L, V0Vector, xrVector, bVector] / fp1}, {x, 0, 5, 0.01}];
```

RxPlot[Vvac,q] is the plot of R(x) for the case with $V_{\text{vac}} = V_{\text{vac}}$ and $b=q$ (in this case $V_{\text{vac}}=50$ and $q=1/6$)

```
In[69]:= RxPlot[Vvac, q] = ListPlot[RxVector, Frame → True,
FrameLabel → {"x", "|Ψ(x)|²/|Ψ(0)|²"}, PlotRange → All,
PlotLabel → StringJoin[{"V_{\text{vac}}=", ToString[Vvac], ", b=", "1/96"}]],
PlotStyle → Blue, Joined → True]
```



R(x) for $V_{\text{vac}}=90$

```
In[70]:= Vvac = 90;
V0Vector[[1]] = Vvac; V0Vector[[Length[xrVector]]] = Vvac;
```

The vector of the first Nterms energies E_n is calculated by means of the function EnV

```
In[72]:= Energies = EnV[Nterms, L, V0Vector, xrVector, bVector]
Out[72]= {7.03073, 7.05529, 7.20151, 7.42232, 7.71797, 8.07402, 8.47005, 8.87814, 9.26319,
9.58352, 9.79776, 28.3977, 28.4456, 29.1836, 30.0616, 31.2144, 32.5777,
34.0707, 35.6059, 37.0712, 38.3216, 39.1826, 63.9642, 63.9853, 66.8904,
68.7485, 71.2101, 74.0519, 77.1218, 80.2789, 83.3539, 86.0926, 88.0992,
98.2257, 98.2268, 111.988, 111.997, 121.528, 124.117, 127.37, 130.498, 133.096,
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194.121, 196.803, 201.015, 206.313, 212.275, 218.611, 225.097, 231.507,
237.518, 242.622, 246.095, 247.078, 281.085, 281.575, 287.516, 293.764,
300.973, 308.706, 316.629, 324.462, 331.841, 338.361, 343.957, 349.053,
353.47, 387.807, 388.265, 398.16, 406.088, 415.011, 424.303, 433.589, 442.524,
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537.999, 548.495, 559.253, 569.827, 579.922, 589.485, 598.869, 608.534,
618.439, 627.473, 659.882, 660.687, 678.197, 689.412, 701.509, 713.694,
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```

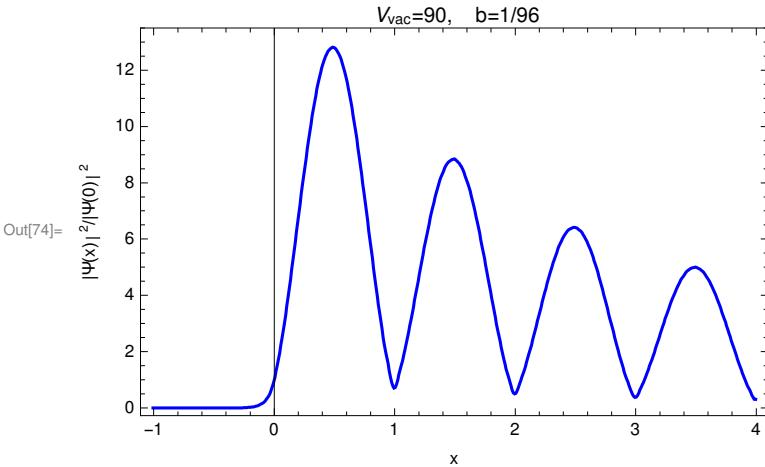
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RxVector is a table of {x,R(x)} with x=0.01, 0.02,...,5

```
In[73]:= xsLeft = 1 + q / 2;
fp1 = ProbDensity[1, xsLeft, Nterms, Nterms, L, V0Vector, xrVector, bVector];
RxVector = Table[{x - xsLeft, ProbDensity[1, x, Nterms,
Nterms, L, V0Vector, xrVector, bVector] / fp1}, {x, 0, 5, 0.01}];
```

RxPlot[Vvac,q] is the plot of R(x) for the case with $V_{vac} = Vvac$ and $b=q$ (in this case $Vvac=50$ and $q=1/6$)

```
In[74]:= RxPlot[Vvac, q] = ListPlot[RxVector, Frame → True,
  FrameLabel → {"x", "|Ψ(x)|²/|Ψ(0)|²"}, PlotRange → All,
  PlotLabel → StringJoin[{" Vvac=", ToString[Vvac], ", b=", "1/96"}],
  PlotStyle → Blue, Joined → True]
```



R(x) for $V_{\text{vac}}=100$

```
In[75]:= Vvac = 100;
V0Vector[[1]] = Vvac; V0Vector[[Length[xrVector]]] = Vvac;
```

The vector of the first Nterms energies E_n is calculated by means of the function EnV

```
In[77]:= Energies = EnV[Nterms, L, V0Vector, xrVector, bVector]
Out[77]= {7.04969, 7.08482, 7.222, 7.44136, 7.73516, 8.08888, 8.48212, 8.8871, 9.26893,
  9.58635, 9.79851, 28.5374, 28.6158, 29.249, 30.1309, 31.2799, 32.6356, 34.1193,
  35.6436, 37.0967, 38.335, 39.1864, 64.6434, 64.6885, 66.9787, 68.889, 71.3637,
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```

```

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7984.72, 8026.68, 8069.23, 8112.53, 8156.58, 8201.27, 8246.87, 8295.05,
8319.74, 8381.04, 8425.29, 8468.56, 8511.9, 8555.11, 8598.61, 8642.72,
8687.6, 8733.22, 8779.47, 8826.29, 8875.81, 8902.15, 8964.73, 9010.49,
9055.83, 9100.75, 9145.75, 9190.92, 9237.17, 9283.75, 9332.92, 9381.14}

```

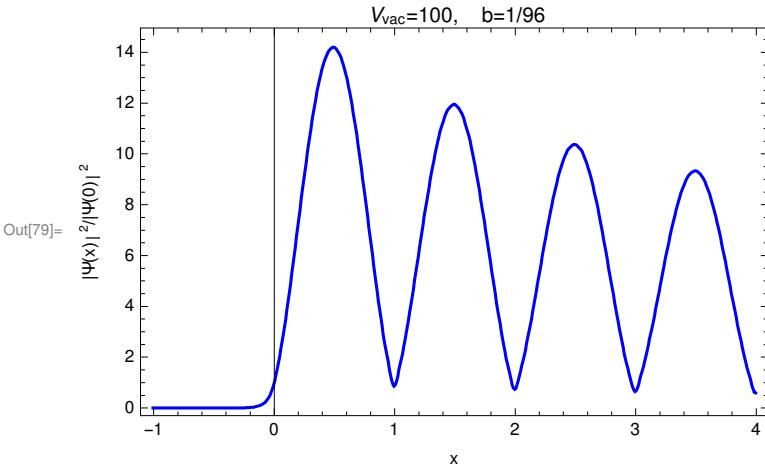
RxVector is a table of {x,R(x)} with x=0.01, 0.02,...,5

```

In[78]:= xsLeft = 1 + q / 2;
fp1 = ProbDensity[1, xsLeft, Nterms, Nterms, L, V0Vector, xrVector, bVector];
RxVector = Table[{x - xsLeft, ProbDensity[1, x, Nterms,
Nterms, L, V0Vector, xrVector, bVector ] / fp1}, {x, 0, 5, 0.01}];
```

RxPlot[Vvac,q] is the plot of R(x) for the case with $V_{vac} = V_{vac}$ and $b=q$ (in this case $V_{vac}=50$ and $q=1/6$)

```
In[79]:= RxPlot[Vvac, q] = ListPlot[RxVector, Frame → True,
  FrameLabel → {"x", "|Ψ(x)|²/|Ψ(0)|²"}, PlotRange → All,
  PlotLabel → StringJoin[{" Vvac=", ToString[Vvac], ", b=", "1/96"}],
  PlotStyle → Blue, Joined → True]
```



R(x) for $V_{vac}=110$

```
In[80]:= Vvac = 110;
V0Vector[[1]] = Vvac; V0Vector[[Length[xrVector]]] = Vvac;
```

The vector of the first Nterms energies E_n is calculated by means of the function EnV

```
In[82]:= Energies = EnV[Nterms, L, V0Vector, xrVector, bVector]
Out[82]= {7.06186, 7.10707, 7.24146, 7.45938, 7.7514, 8.10289, 8.4935, 8.89552, 9.27433,
  9.58901, 9.79922, 28.6302, 28.7446, 29.3199, 30.199, 31.3424, 32.6902, 34.1646,
  35.6785, 37.1203, 38.3473, 39.1898, 65.1269, 65.2105, 67.0849, 69.0288,
  71.5056, 74.3389, 77.3885, 80.5139, 83.5419, 86.2127, 88.1398, 112.188,
  112.198, 121.036, 121.743, 122.8, 125.361, 129.088, 133.425, 138.031, 142.621,
  146.709, 149.553, 151.608, 154.175, 156.829, 179.167, 179.182, 194.239,
  197.759, 201.896, 205.856, 209.891, 214.71, 220.423, 226.666, 233.07, 239.235,
  244.443, 255.624, 255.707, 284.738, 287.223, 291.015, 296.404, 303.095,
  310.623, 318.609, 326.792, 334.921, 342.651, 349.396, 354.209, 355.709,
  393.07, 394.699, 400.43, 408.156, 416.989, 426.398, 436.054, 445.68, 454.933,
  463.348, 470.541, 476.597, 481.565, 520.045, 521.401, 530.189, 539.903,
  550.532, 561.588, 572.7, 583.548, 593.781, 603.186, 611.984, 620.552, 628.251,
  666.171, 667.588, 679.78, 691.307, 703.65, 716.24, 728.717, 740.761, 752.156,
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  1247.17, 1263.76, 1280.79, 1297.73, 1314.26, 1330.29, 1346.04, 1361.95,
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  1695.97, 1724.04, 1743.88, 1763.93, 1783.74, 1803.07, 1822.02, 1840.91,
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7517.21, 7558.87, 7601.29, 7644.4, 7688.17, 7735.76, 7759.08, 7817.26, 7860.24,
7902.67, 7944.64, 7986.4, 8028.33, 8070.8, 8114.01, 8157.99, 8202.63, 8248.28,
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8689.08, 8734.63, 8780.86, 8827.73, 8877.98, 8904.07, 8965.86, 9011.86,
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```

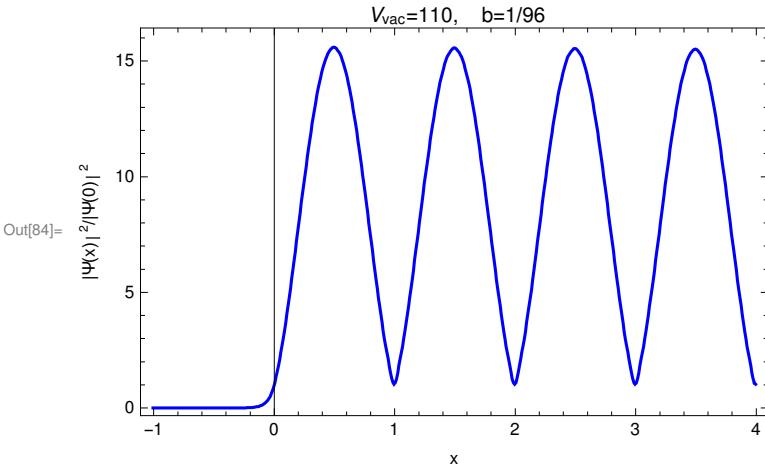
RxVector is a table of {x,R(x)} with x=0.01, 0.02,...,5

```

In[83]:= xsLeft = 1 + q / 2;
fp1 = ProbDensity[1, xsLeft, Nterms, Nterms, L, V0Vector, xrVector, bVector];
RxVector = Table[{x - xsLeft, ProbDensity[1, x, Nterms,
Nterms, L, V0Vector, xrVector, bVector] / fp1}, {x, 0, 5, 0.01}];
```

RxPlot[Vvac,q] is the plot of R(x) for the case with $V_{vac} = V_{vac}$ and $b=q$ (in this case $V_{vac}=50$ and $q=1/6$)

```
In[84]:= RxPlot[Vvac, q] = ListPlot[RxVector, Frame → True,
  FrameLabel → {"x", "|Ψ(x)|²/|Ψ(0)|²"}, PlotRange → All,
  PlotLabel → StringJoin[{" Vvac=", ToString[Vvac], ", b=", "1/96"}],
  PlotStyle → Blue, Joined → True]
```



R(x) for Vvac=120

```
In[85]:= Vvac = 120;
V0Vector[[1]] = Vvac; V0Vector[[Length[xrVector]]] = Vvac;
```

The vector of the first Nterms energies E_n is calculated by means of the function EnV

```
In[87]:= Energies = EnV[Nterms, L, V0Vector, xrVector, bVector]
Out[87]= {7.07002, 7.12415, 7.25908, 7.47614, 7.76665, 8.1161, 8.50424, 8.90348, 9.27943,
  9.59152, 9.79989, 28.6922, 28.8436, 29.391, 30.2645, 31.4017, 32.7416,
  34.2072, 35.7111, 37.1421, 38.3587, 39.193, 65.4692, 65.6068, 67.2085,
  69.1673, 71.6387, 74.461, 77.4962, 80.6036, 83.6089, 86.2521, 88.1522, 115.23,
  115.258, 121.733, 124.643, 128.039, 130.253, 131.566, 134.385, 138.75,
  143.497, 148.215, 152.56, 156.043, 158.482, 158.717, 183.692, 183.723,
  194.51, 198.589, 203.562, 208.543, 212.98, 217.157, 221.95, 227.615, 233.724,
  239.698, 244.668, 260.81, 260.864, 285.384, 289.099, 293.471, 298.514,
  304.623, 311.761, 319.562, 327.705, 335.915, 343.871, 351.012, 358.241,
  358.821, 394.513, 397.283, 402.405, 409.606, 418.148, 427.444, 437.126,
  446.92, 456.535, 465.534, 473.241, 478.909, 482.461, 522.203, 524.36, 531.644,
  541.096, 551.621, 562.696, 573.953, 585.095, 595.756, 605.521, 614.195,
  622.026, 628.769, 668.72, 670.728, 680.965, 692.409, 704.744, 717.432,
  730.124, 742.495, 754.223, 765.164, 775.593, 785.832, 795.171, 834.675,
  836.777, 850.245, 863.521, 877.506, 891.721, 905.781, 919.385, 932.34,
  944.771, 957.114, 969.596, 981.271, 1020.12, 1022.52, 1039.25, 1054.28,
  1069.91, 1085.6, 1101.02, 1115.91, 1130.24, 1144.3, 1158.56, 1173.12, 1187.01,
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  1363.68, 1379.79, 1396.32, 1412.34, 1450.21, 1453.61, 1476.61, 1495., 1513.74,
  1532.32, 1550.48, 1568.11, 1585.43, 1602.86, 1620.79, 1639.19, 1657.29,
  1694.94, 1699., 1724.96, 1744.98, 1765.19, 1785.21, 1804.76, 1823.84, 1842.71,
  1861.82, 1881.5, 1901.73, 1921.82, 1959.35, 1964.24, 1992.94, 2014.58,
```

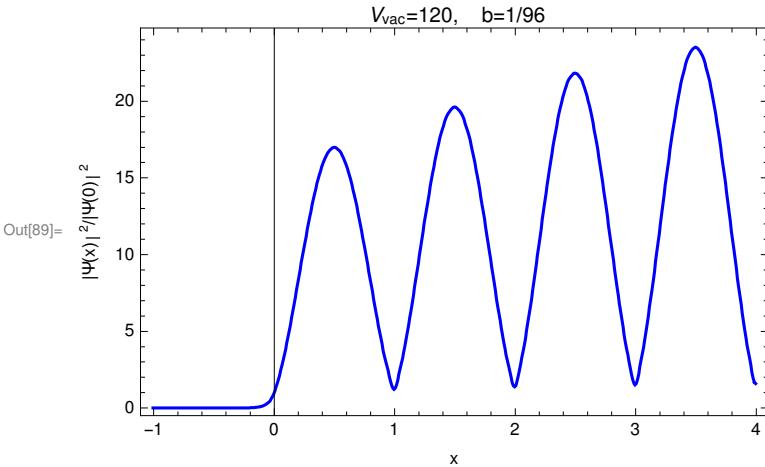
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RxVector is a table of {x,R(x)} with x=0.01, 0.02,...,5

```
In[88]:= xsLeft = 1 + q / 2;
fp1 = ProbDensity[1, xsLeft, Nterms, Nterms, L, V0Vector, xrVector, bVector];
RxVector = Table[{x - xsLeft, ProbDensity[1, x, Nterms,
Nterms, L, V0Vector, xrVector, bVector] / fp1}, {x, 0, 5, 0.01}];
```

RxPlot[Vvac,q] is the plot of R(x) for the case with $V_{vac} = Vvac$ and $b=q$ (in this case $Vvac=50$ and $q=1/6$)

```
In[89]:= RxPlot[Vvac, q] = ListPlot[RxVector, Frame → True,
  FrameLabel → {"x", "|\Psi(x)|^2/|\Psi(0)|^2"}, PlotRange → All,
  PlotLabel → StringJoin[{" Vvac=", ToString[Vvac], ", b=", "1/96"}],
  PlotStyle → Blue, Joined → True]
```



R(x) for Vvac=150

```
In[90]:= Vvac = 150;
V0Vector[[1]] = Vvac; V0Vector[[Length[xrVector]]] = Vvac;
```

The vector of the first Nterms energies E_n is calculated by means of the function EnV

```
In[92]:= Energies = EnV[Nterms, L, V0Vector, xrVector, bVector]
Out[92]= {7.08309, 7.15689, 7.30016, 7.51846, 7.80653, 8.15128, 8.53311, 8.92496, 9.29322,
  9.59832, 9.80171, 28.7859, 29.0316, 29.5749, 30.4381, 31.5599, 32.8792,
  34.3209, 35.7979, 37.2002, 38.3887, 39.2014, 65.9957, 66.344, 67.6233,
  69.5592, 71.9953, 74.7775, 77.7679, 80.8237, 83.7684, 86.343, 88.1798,
  118.775, 118.949, 122.264, 125.603, 129.68, 134.223, 139.028, 143.933,
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  973.744, 982.954, 1026.81, 1031.29, 1043.42, 1057.96, 1073.49, 1089.42,
  1105.39, 1121.06, 1136.12, 1150.34, 1163.81, 1176.78, 1188.64, 1232.53,
  1237.08, 1251.89, 1268.38, 1285.61, 1303.1, 1320.48, 1337.45, 1353.75,
  1369.37, 1384.61, 1399.76, 1414.04, 1457.67, 1462.54, 1480.18, 1498.52,
  1517.42, 1536.43, 1555.2, 1573.48, 1591.12, 1608.27, 1625.34, 1642.55,
  1659.12, 1702.55, 1707.85, 1728.36, 1748.47, 1768.95, 1789.43, 1809.59,
  1829.22, 1848.27, 1867.01, 1885.88, 1905.07, 1923.81, 1967., 1972.92, 1996.27,
```

```

2018.09, 2040.17, 2062.12, 2083.67, 2104.68, 2125.2, 2145.56, 2166.22,
2187.31, 2208.15, 2251.29, 2257.9, 2284.04, 2307.53, 2331.11, 2354.51,
2377.47, 2399.88, 2421.9, 2443.9, 2466.31, 2489.24, 2512.09, 2555.2, 2562.67,
2591.48, 2616.61, 2641.76, 2666.61, 2690.98, 2714.83, 2738.37, 2762.01,
2786.15, 2810.87, 2835.71, 2878.98, 2887.36, 2918.77, 2945.52, 2972.13,
2998.43, 3024.23, 3049.54, 3074.62, 3099.89, 3125.72, 3152.2, 3178.92, 3222.41,
3231.84, 3265.71, 3294.06, 3322.22, 3349.97, 3377.2, 3403.99, 3430.62,
3457.52, 3485.04, 3513.23, 3541.83, 3585.72, 3596.23, 3632.52, 3662.44,
3692.04, 3721.25, 3749.92, 3778.19, 3806.38, 3834.91, 3864.08, 3893.97,
3924.35, 3968.67, 3980.39, 4018.95, 4050.44, 4081.58, 4112.24, 4142.38,
4172.15, 4201.9, 4232.03, 4262.86, 4294.41, 4326.59, 4371.53, 4384.44, 4425.26,
4458.3, 4490.87, 4522.98, 4554.58, 4585.86, 4617.17, 4648.91, 4681.36, 4714.57,
4748.44, 4794., 4808.26, 4851.16, 4885.75, 4919.87, 4953.45, 4986.53, 5019.31,
5052.19, 5085.53, 5119.6, 5154.44, 5190.06, 5236.41, 5251.95, 5296.99, 5333.11,
5368.63, 5403.67, 5438.23, 5472.53, 5506.96, 5541.9, 5577.58, 5614.04,
5651.28, 5698.4, 5715.38, 5762.37, 5800.02, 5837.1, 5873.62, 5909.66, 5945.49,
5981.48, 6018.01, 6055.29, 6093.35, 6132.32, 6180.37, 6198.68, 6247.73,
6286.9, 6325.35, 6363.34, 6400.87, 6438.22, 6475.77, 6513.88, 6552.75, 6592.41,
6632.92, 6681.87, 6701.71, 6752.57, 6793.27, 6833.32, 6872.79, 6911.82,
6950.69, 6989.8, 7029.48, 7069.94, 7111.19, 7153.44, 7203.47, 7224.6, 7277.52,
7319.74, 7361.09, 7402.04, 7442.55, 7482.95, 7523.62, 7564.88, 7606.92,
7649.75, 7693.46, 7744.49, 7767.21, 7821.84, 7865.56, 7908.58, 7951.02,
7993.05, 8034.98, 8077.2, 8120.02, 8163.64, 8208.03, 8253.65, 8305.94,
8329.72, 8386.68, 8431.97, 8476.04, 8519.96, 8563.45, 8606.91, 8650.69,
8695.13, 8740.33, 8786.35, 8833.24, 8886.69, 8912.1, 8970.53, 9017.28,
9063.37, 9108.85, 9154.32, 9199.5, 9246.21, 9292.45, 9345.45, 9393.76}

```

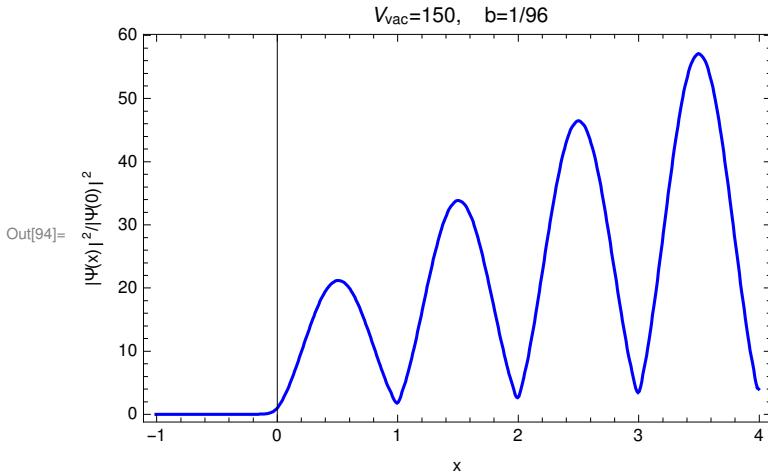
RxVector is a table of {x,R(x)} with x=0.01, 0.02,...,5

```

In[93]:= xsLeft = 1 + q / 2;
fp1 = ProbDensity[1, xsLeft, Nterms, Nterms, L, V0Vector, xrVector, bVector];
RxVector = Table[{x - xsLeft, ProbDensity[1, x, Nterms,
Nterms, L, V0Vector, xrVector, bVector] / fp1}, {x, 0, 5, 0.01}];
```

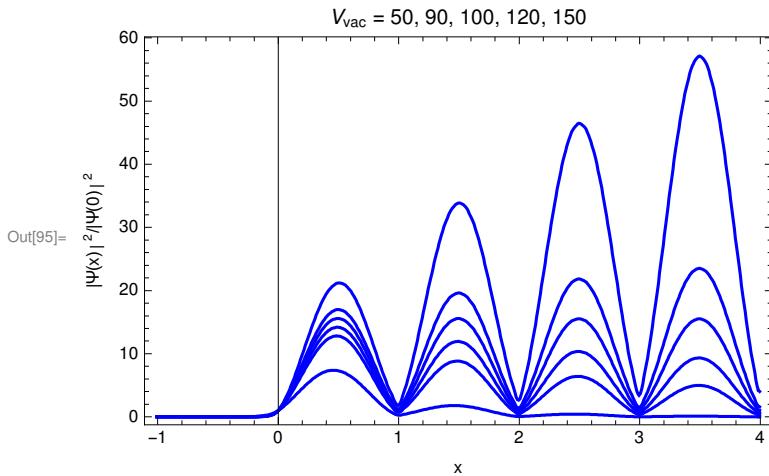
RxPlot[Vvac,q] is the plot of R(x) for the case with $V_{vac} = Vvac$ and $b=q$ (in this case $Vvac=50$ and $q=1/6$)

```
In[94]:= RxPlot[Vvac, q] = ListPlot[RxVector, Frame → True,
  FrameLabel → {"x", "|Ψ(x)|²/|Ψ(0)|²"}, PlotRange → All,
  PlotLabel → StringJoin[{" Vvac=", ToString[Vvac], ", b=", "1/96"}],
  PlotStyle → Blue, Joined → True]
```

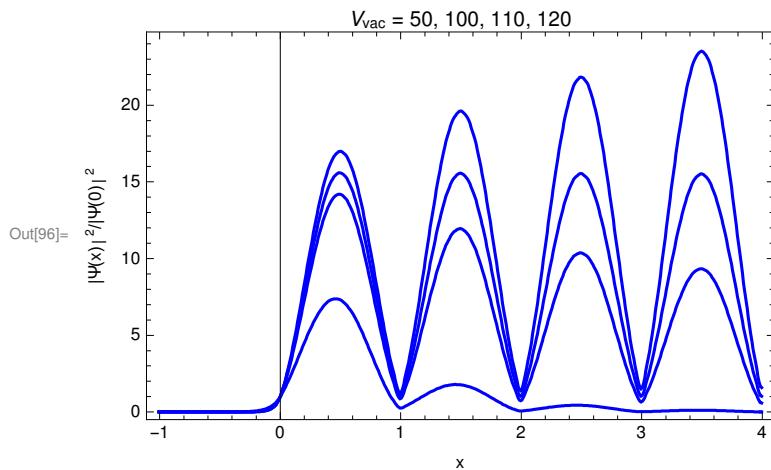


Comparison of $R(x)$ for several value of V_{vac}

```
In[95]:= Show[{RxPlot[50, q], RxPlot[90, q], RxPlot[100, q],
  RxPlot[110, q], RxPlot[120, q], RxPlot[150, q]}, PlotRange → All,
  PlotLabel → StringJoin[{"Vvac = 50, 90, 100, 120, 150"}]]
```



```
In[96]:= Show[{RxPlot[50, q], RxPlot[100, q], RxPlot[110, q], RxPlot[120, q]}, PlotRange -> All, PlotLabel -> StringJoin[{"Vvac = 50, 100, 110, 120"}]]
```



```
In[97]:= Show[{RxPlot[50, q], RxPlot[100, q], RxPlot[120, q]}, PlotRange -> All, PlotLabel -> StringJoin[{"Vvac = 50, 100, 120"}]]
```

